Modelling Electricity Auctions

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Key Issues in Reforming Electricity Wholesale Markets

I. Market Structure & Market Power in Generation

- E&W – a virtual duopoly at privatisation led to 10 years of market power problems
- Australia – asset divestures created more competitive markets at outset
- Spain (Endesa and Iberdrola with 80% of market, Union Fenosa with 15%)
- California – debate/litigation over whether market melt down caused by market rules or market power, or both

II. Market "Architecture" and "Auction Design" Issues

- market "architecture" refers to structure of wholesale market, i.e. a single, "centralised" spot market or series of forward and spot markets, e.g. E&W, California
- market "design" refers to detailed design of electricity auctions e.g. E&W, California

Key Lesson from Experience – getting I right makes II much less important?

"Centralised" Versus "Decentralised" Markets

E&W (old), Australia, Spain, … introduced *electricity pools:*

- c*entralised* spot market for the wholesale exchange of electricity:
- all supply and demand must go through the pool
- no <u>physical</u> bilateral trades

Pools overcome classic *co-ordination* problem between generation and transmission:

- grid must match supply and demand continuously
- requires minute-by-minute coordination of transmission and generation

Pool *optimisation problem* is to *minimise* total cost of meeting demand subject to:

- generation costs of different technologies
- transmission constraints and losses

Mimics pre-reform 'command and control' organisation

“Decentralised” Electricity Markets

California, NETA (England)

- a series of voluntary forward markets and spot markets
- bilateral trading permitted
- spot or balancing market mandatory for “out of balance” suppliers (i.e. to coordinate supply and demand)

Advocates of “decentralised” design (e.g. Wilson, 2002) argue that:

- optimisation in pools is illusory as bids do not reveal costs
- sequence of markets increases competition/mitigates market power
- opponents argue it is more opaque and more opportunities for strategic behaviour
- more likely to result in productive inefficiency (bilateral trades)

Conclusion: The jury is still out on this issue

Design Issues: Auction Format

California/UK debate over auction format:

Uniform (First-Price) Auction:

- each seller receives bid which just clears the market (i.e. price bid of marginal successful unit or last unit sold)

Discriminatory Auction:

- each seller receives own bid price for each unit sold

2nd Price (Vickrey) Auction:

- each seller receives bid prices of unsuccessful bids he replaces, or
- market-clearing price ignoring that bidder's own bids, when there are no unsuccessful (replaced) bids.

Argument was that discriminatory auctions would perform better than standard uniform auction – opposite of Treasury auction debate.

Uniform Versus Discriminatory

Properties of standard - e.g. uniform, discriminatory - auctions:

- a seller's bid determines the price received with some probability, hence they have an incentive to increase bids above cost
- units may not be sold by cheapest suppliers - so inefficient allocation of resources results
- hence we would expect **above-cost bidding** and **inefficient despatch** whenever generators have market power
- in uniform auction, large suppliers have incentives to increase their bid as marginal bid determines price paid on submarginal units (*Ausubel and Cramton, "Demand Reduction and Inefficiency…"*)
- in discriminatory auction, each trader tries to guess market-clearing price (or plays a mixed strategy)

Conclusion: No general results on whether uniform preferred to discriminatory

Vickrey Auctions

Properties of Vickrey Auctions

- weakly dominant strategy to bid true reservation price (costs of supply)
- units allocated to (sold by) the lowest cost suppliers, i.e. efficient resource allocation achieved

But:

- cannot use "truthful" bids to remunerate generators - *they would then alter their bids*
- generators receive the opportunity costs of their bids, i.e. the additional cost of supplying the market without their bids
- with capacity-constrained suppliers these payments can be large

Conclusion: Efficiency achieved but costs can be high

NETA

England and Wales reforms (2001) changed market in both dimensions:

- argued that uniform auction paid generators too much for bids below market-clearing price (contrast to "Friedman" mistake)
- "Klemperer" argument that uniform auctions led to "collusive-like" behaviour
- from centralised pool to decentralised series of forward markets plus near real-time balancing market (approx. 2%-4% of traded electricity)
- from uniform to discriminatory auction for balancing market

Has NETA worked?

"NETA reforms – alongside other factors such as falling fuel prices, a generous capacity margin and increased competition in generation – have resulted in a 40 per cent reduction in the costs of wholesale electricity since 1998, when NETA reforms were first proposed by Government." *Ofgem, 2002*

Impact of NETA on Prices

Since NETA "Go-Live" (March 2001):

- baseload prices fell by 20% and peak prices by 27% in the first year.
- domestic customers' bills fell by 4%-8% depending upon the supplier
- wholesale prices have since risen back to pre-reform levels

Evans and Green (2003)

- attributed price falls to decline in collusive behaviour brought about by new market design

Newbery (2003), Bower (2002)

- changes in market structure and capacity margins were key factor

Fabra and Toro (2003)

- found both NETA and HHI/capacity margin were significant in explaining decline in Lerner index

Evolution of Market Structure in E&W, 1990- 2002

Capacity Shares (%)	**1990/91**	**1995/6**	**1997/98**	**1998/99**	**2001/02**
NP (Innogy)	48%	34%	27%	24%	13%
PowerGen	30%	29%	23%	24%	12%
East/TXU	_	_	12.5%	11%	10%
BE	14%	18%	17%	11%	14%
IPP	_	6%	6%	15%	21%
AES	_	_	_	1%	7%
EME	_	_	_	4%	9%
I'connectors	4.5%	6%	5.5%	5%	5%
Other	3.5%	7%	9%	5%	9%
TOTAL	100%	100%	100%	100%	100%
HHI's	***3432***	***2442***	***1850***	***1686***	***1286***

Modelling Electricity Auctions

Fabra, von der Fehr and Harbord (2003)(2004):

- attempt to address the *auction format* issue in a tractable model, i.e. uniform versus discriminatory
- followed 'multi-unit auction approach' of von der Fehr and Harbord (1993)

Modelling Choices:

1. Number of bids submitted by each supplier: finite or continuous?
2. Bids can be short-lived or long-lived
3. Firms are capacity-constrained

Finite Number of Bids

- E&W – 3 incremental bid prices per capacity unit
- Spain – up to 25 price-quantity pairs per company
- i.e. *discrete, multi-unit auctions*, not a *continuous share auctions*

Finite Number of Bids (cont.)

Does finite/continuous distinction matter?

Short-Lived Bids: Demand is known when bids are submitted

- continuous uniform auction yields 'collusive-like' equilibria with no capacity constraints
- firms care about single point on supply curve, corresponding to market-clearing price
- can submit "steep" supply curve, making residual demand faced by rival "steep" also
- cost of undercutting is made high (i.e. "price effect" > "quantity effect")
- in discriminatory auction firms care about all bids - not an equilibrium strategy
- in discrete auction undercutting slightly is always worthwhile ("quantity effect" > "price effect")
- collusive equilibria don't exist

Finite Number of Bids (cont.)

Long-Lived Bids: Fixed over many periods - creates demand uncertainty or volatility

Klemperer and Meyer (1989) "Supply Function" Model

- rules out many equilibria in continuous auction - fewer pay-off irrelevant bids
- yields continuum of pure-strategy equilibria with prices between perfectly competitive and Cournot outcomes
- discrete multi-unit model has only mixed strategy equilibria
- equilibria do not converge to continuous model as bid step becomes small
- hence continuous model does not approximate discrete model in the limit (Kremer and Nyborg, 2003)
- continuous models ignore capacity constraints which determine who has market power when, i.e. each firm's residual demand

Conclusion: Discrete, multi-unit auction model with capacity-constrained firms

Basic Duopoly Model

Two independent suppliers $i = 1; 2$ with productive capacities $k_i > 0$

Costs

- constant unit costs, with $c_1=0$, $c_2 = c > 0$, i.e. flat cost curves

Demand

- $\theta \in (0, k_1+k_2)$ is completely price inelastic

Timing

- having observed the realization of demand, suppliers simultaneously submit price offers $b_i < \boldsymbol{P}$ for their entire capacities
- $\boldsymbol{P}$ is the maximum willingness to pay (VOLL) or maximum allowable price, fixed by regulation

Basic Duopoly Model (cont.)

Outputs

- determined by ranking of prices offers and independent of the auction format
- output allocated to supplier i, $i = 1, 2$, denoted by $q_i\ (\theta; \mathbf{b})$, is given by

$$
q_i\ (\theta; \mathbf{b}) = \begin{cases} \min\ \{\theta,\ k_i\} \text{ if } b_i < b_j \\ \rho_i \min\ \{\theta,\ k_i\} + [1 - \rho_i] \max\ \{0,\ \theta - k_j\} \text{ if } b_i = b_j \\ \max\ \{0,\ \theta - k_j\} \text{ if } b_i > b_j\ , \end{cases}
$$

where $\rho_i + \rho_j = 1$, $\rho_i = 1$ *if* $c_i < c_j$ *and* $\rho_i = \frac{1}{2}$ *if* $c_i = c_j$

Basic Duopoly Model (cont.)

Payments

Uniform auction:
All suppliers are paid the highest accepted bid (i.e. system marginal price):

$$\pi_i\,(\theta;\,\mathbf{b}) \quad =[b_j - c_i]\; q_i(\theta;\,\mathbf{b}),\text{ if } b_i \le b_j \text{ and } \theta > k_i$$
$$=[b_i - c_i]\; q_i(\theta;\,\mathbf{b}),\text{ otherwise}$$

where $q_i\,(\theta;\,\mathbf{b})$ is determined as above.

Discriminatory auction:
Suppliers are paid their own bid:

$$\pi_i\,(\theta;\,\mathbf{b}) = \;[b_i - c_i]\; q_i\,(\theta;\,\mathbf{b})$$

where $q_i\,(\theta;\,\mathbf{b})$ is determined as above

.

Equilibrium Analysis

Lemma 1: *In any pure-strategy equilibrium, the highest accepted price offer equals either c or P.*

Proposition 1: *There exists* $\theta^* = \theta^*(c; k_1; k_2; P)$ *such that:*

(i) *(Low demand): If* $\theta < \theta^*$*, in the unique pure-strategy equilibrium the highest accepted offer price equals c.*

(ii) *(High demand): If* $\theta > \theta^*$*, all suppliers are paid prices that exceed c:*

- *a pure-strategy equilibrium exists in the uniform auction, with the highest accepted offer price equal to P*
- *in the discriminatory auction only a mixed strategy equilibrium exists.*

Comparison Across Auctions

Low Demand State [$\theta < \theta^*$]

Bidding: 'Bertrand-like' competitive bidding with both firms bidding $b_i = c = SMP$.

Revenues: $R_u = R_d = c\theta$

Cost efficiency: $C_u = C_d = 0$ (low-cost producer supplies all demand)

Determining θ*: If $k_2 > k_1$, $\theta^* = k_1$; if $k_1 > k_2$, $\theta^* = \min\{(P/(P-c))k_2, k_1)\})$

High Demand State[$\theta > \theta^*$]

Bidding:

Uniform: Either $b_1 < b_2 = P$ or $b_2 < b_1 = P$

Discriminatory: *Mixed strategy equilibrium*, *with* $c < b_i = P$

Revenues: $R_u > R_d$; $(R_u = P\theta, R_d < P\theta)$

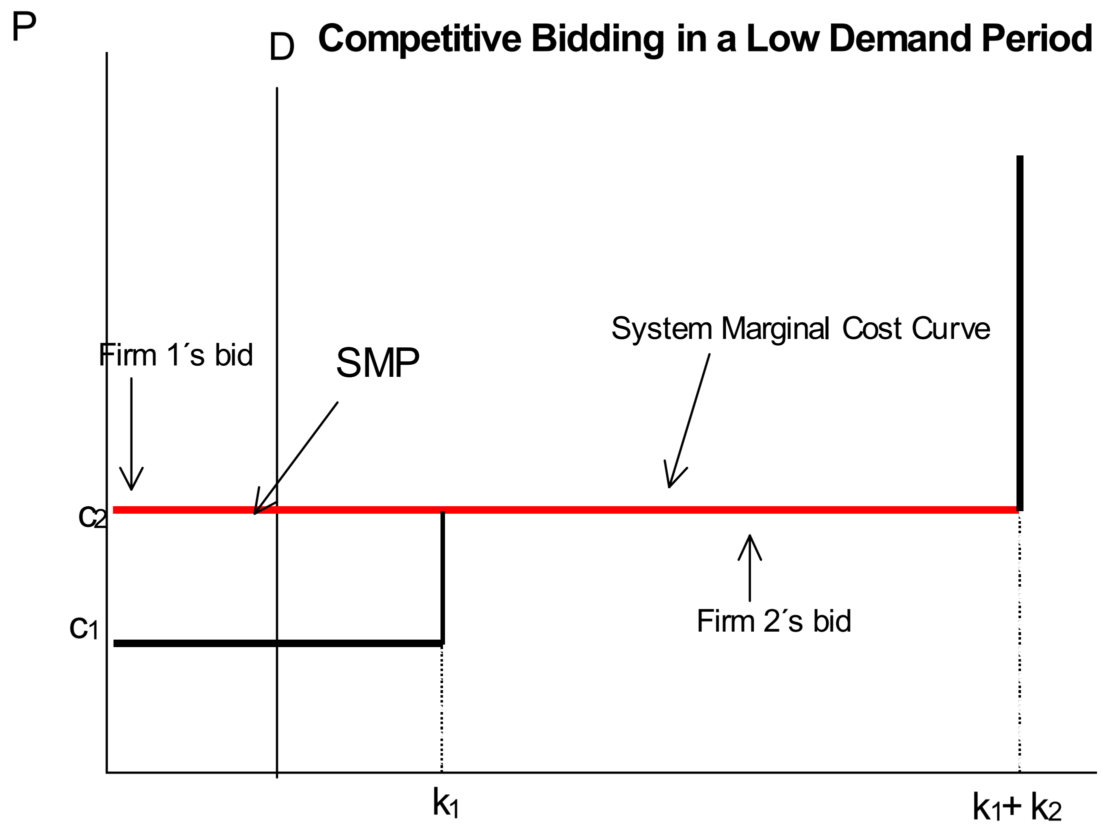
Cost efficiency: $C_u < C_d$ if in the uniform auction the equilibrium with $b_2 > b_1$ is played; $C_u > C_d$ otherwise

Comparison Across Auctions (cont.)

High Demand State[$\theta > \theta^*$]

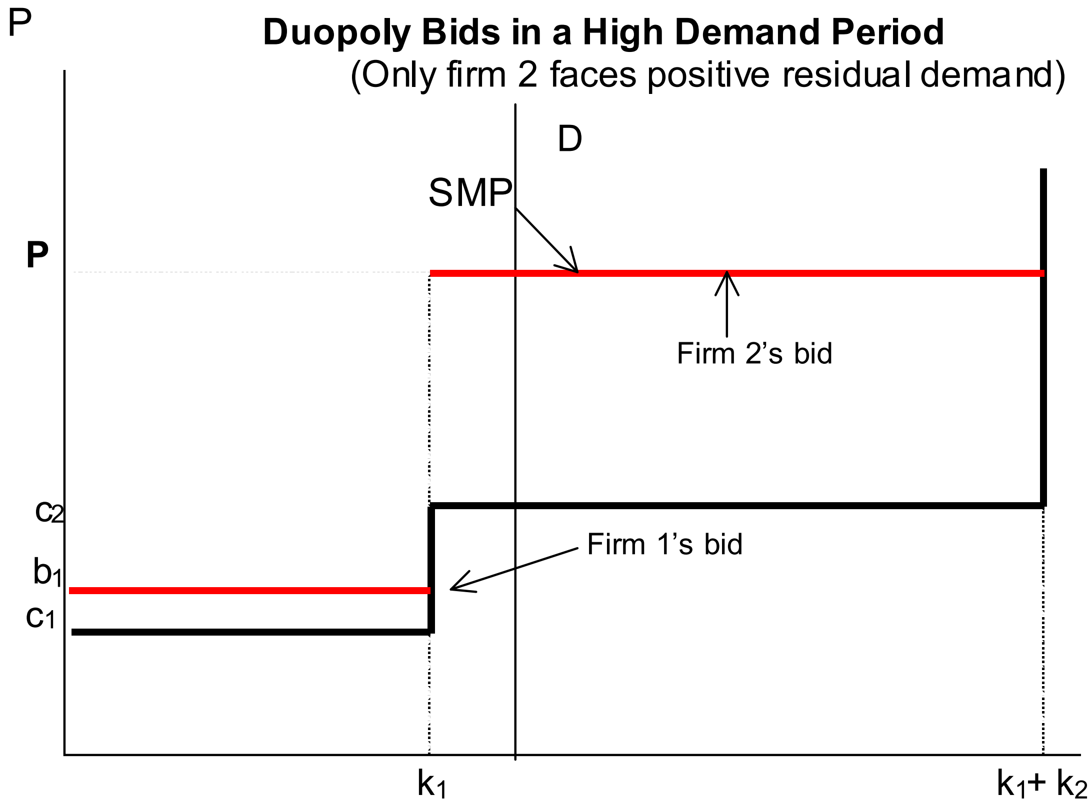
- pure strategy equilibria in uniform auction are very asymmetric
- one firm bids maximum price $\boldsymbol{P}$ while other firm bids low enough so that undercutting is not profitable
- maximum efficiency or inefficiency
- mixed strategy equilibria exist also, but are (weakly) payoff-dominated by pure strategy equilibria
- e.g. when c=0, each firm gets $\pi_i = P[\theta - k_j]$ - the high-bidder's payoff
- in symmetric case, the two auctions are then equivalent
- revenue/efficiency comparison is difficult otherwise

Equilibrium Bids: Low Demand



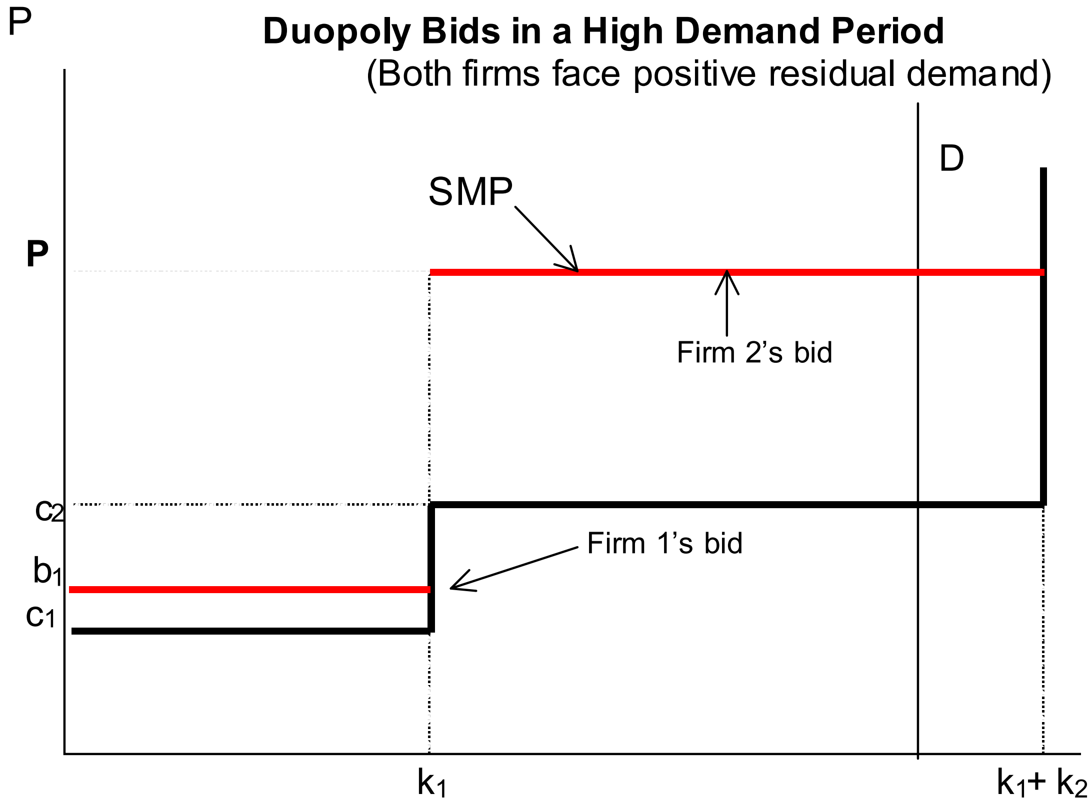
Same result in both auctions

Uniform Auction Bids: High Demand



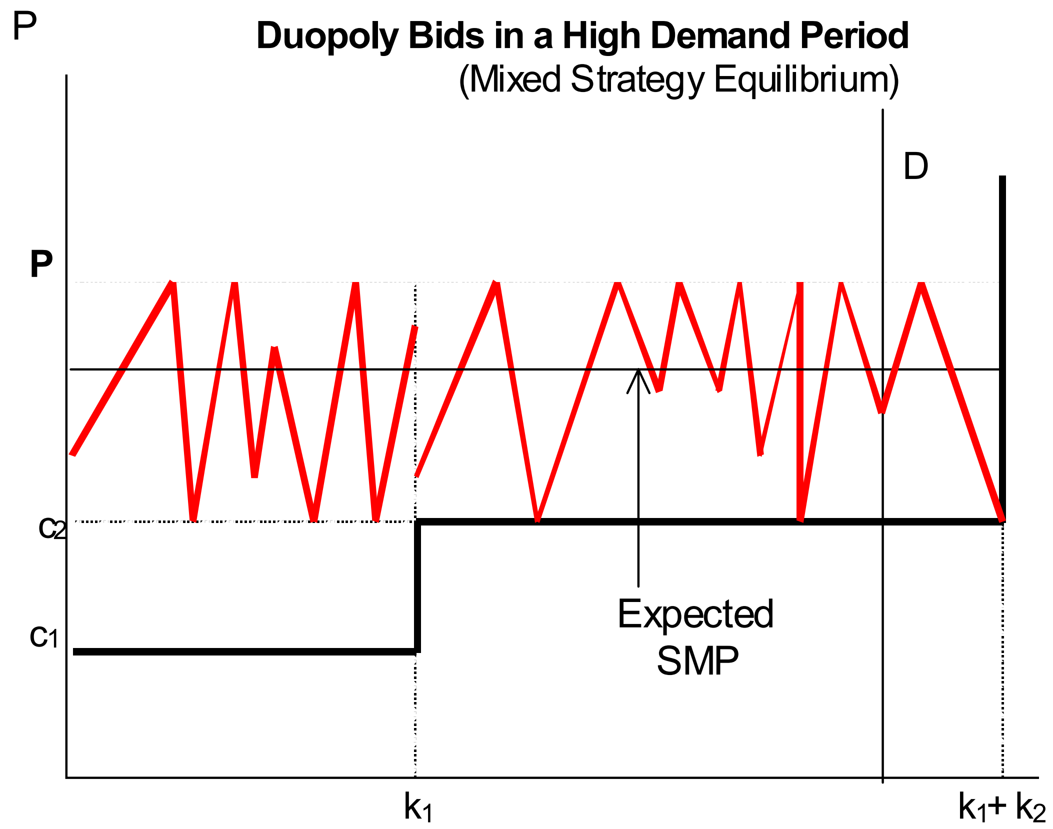
If Firm 1 bids high, produces no output - hence Firm 2 bids high and supplies (θ-k_1)

Uniform Auction Bids: High Demand



Either firm could bid high or low in this case.

Discriminatory Auction Bids: High Demand



Changing θ*

Incidence of *low* versus *high* demand states determines:

- whether the industry is more or less competitive
- how and when outcomes differ in the two types of auction

Low demand states more likely with:

- Larger installed capacity
- Stricter regulation (lower ***P***)
- Capacity symmetry
- Price elastic demand
- Less concentrated market structure

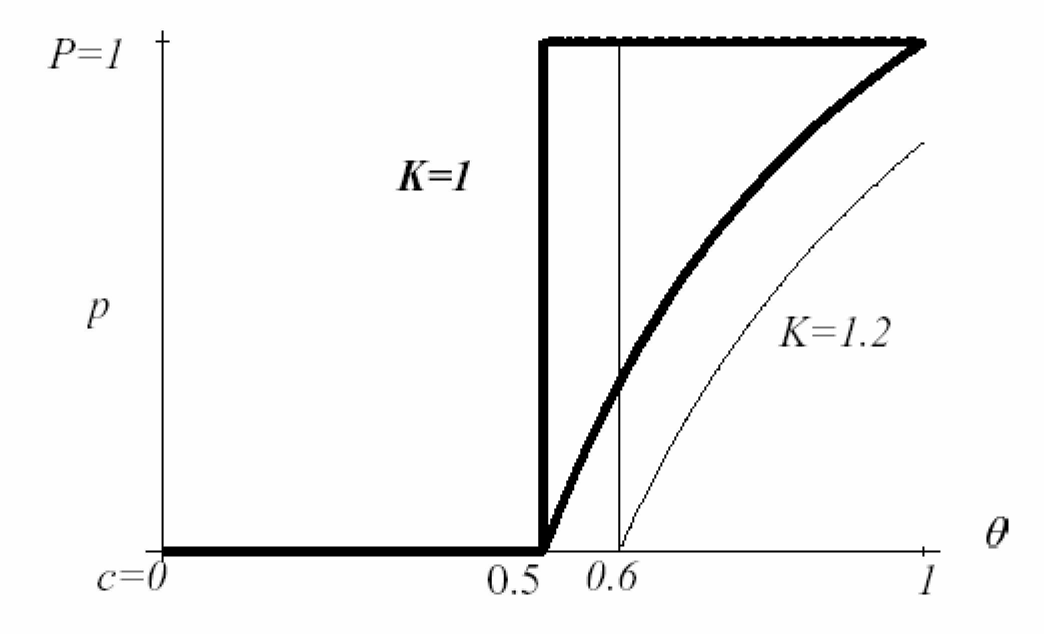
Effects of a change in the incidence of high versus low demand states:

- prices jump down from ***P*** to *c* in the uniform auction
- increases the "intensity of competition" ***within*** high demand states in the discriminatory auction

Increasing Installed Capacity

Example: Uniform demand distribution on [0,1], $k_1 + k_2 = K = 1$, with $k_1 = k_2$. c=0 and $\boldsymbol{P}$=1.

Expected Equilibrium Price for Different Demand Realizations



Comparison depends in general on distribution of demand

Increasing Installed Capacity (cont.)

Example: Expected payments to suppliers taken over all possible demand realizations are: $ER_d = [1-k]^2$ and $ER_u = [1/2][1-k^2]$, respectively.

Table 1

K	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**
$\boldsymbol{ER_d}$	0.25	0.16	0.09	0.04	0.01	0
$\boldsymbol{ER_u}$	0.375	0.320	0.255	0.180	0.0950	0
$\boldsymbol{ER_u/ER_d}$	1.5	2	2.8	4.5	9.5	na

- at $K = 1$, total expected payments are 50% higher in the uniform auction
- a similar reduction in average prices would require an excess capacity of 40% (i.e. $K =$ 1:4) in uniform auction
- in both auctions, increasing the size of the players reduces both average prices and revenues

Increasing Capacity Asymmetry

Example Now assume K=1, with $k_1 = k_2$

Table 2

$\boldsymbol{k_1}$	0.5	0.6	0.7	0.8	0.9	1
$\boldsymbol{k_2}$	0.5	0.4	0.3	0.2	0.1	0
$\boldsymbol{ER_d}$	0.250	0.300	0.350	0.400	0.450	0.5
$\boldsymbol{ER_u}$	0.375	0.420	0.455	0.480	0.495	0.5
$\boldsymbol{ER_u/ER_d}$	1.5	1.4	1.3	1.2	1.1	1

- reducing the size of the smaller supplier increases the incidence of high-demand states
- implies poorer performance in both types of auction
- in the discriminatory auction, the larger supplier faces a larger residual demand and hence has more to gain from submitting higher offer prices. Given this, the smaller supplier responds by increasing its offer prices also.

Reducing the Market Reserve Price

How do changes to the market reserve price *P* affect performance in the two auctions?

Fix total capacity so $K = 1$ and consider symmetric firms, i.e. $k_1 = k_2 = 0.5$.

Table 3

P	**1**	**0.90**	**0.75**	**0.50**	**0.25**	**0**
ER_d	0.250	0.225	0.1875	0.125	0.0625	0
ER_u	0.375	0.3375	0.28125	0.1875	0.09375	0
ER_u/ER_d	1.5	1.5	1.5	1.5	1.5	1.5

- reducing the market reserve price reduces equilibrium price (and hence revenues) in both types of auction
- no effect on relative performance.
- this is because when $c = 0$ equilibrium revenues are proportional to $\boldsymbol{P}$ in both auctions, i.e. $ER_d = P/4$; $ER_u = 3P/8$
- no effect on θ^* when $c=0$

Variations on Model: Multiple Unit Suppliers

Basic model <u>assumes</u> flat bid functions, i.e. a single bid for total capacity:

- equilibria with flat bid functions generalise to multiple bid-step case
- hence equilibrium outcomes (essentially) independent of number of allowable bid-steps
- in discriminatory auction, flat bid function equilibria not unique

Discrete versus Continuous Bidding:

- unique zero profit equilibrium equilibrium outcome in uniform auction, in contrast to continuous supply function models
- discrete bidding performs better (i.e. no collusive outcomes in low demand states)
- reducing the number of steps does not affect the outcomes, but makes bidding simpler

Variations: Uncertain Demand

With long-lived bids firms face 'uncertainty', i.e. bids must cover both low and high demand periods

- demand θ takes values in $(0, k_1 + k_2)$ according to $G(\theta)$
- similar results to above if θ varies *within* a high or low demand state

Lemma 2 *There does not exist an equilibrium in pure strategies in either auction. In the unique mixed-strategy equilibrium suppliers submit bids that strictly exceed c.*

- the two auction formats are equivalent if suppliers are symmetric; the comparison is unclear otherwise.
- with symmetric suppliers, long-lived bids perform better - i.e. same revenues in discriminatory auction and lower revenues in the uniform auction
- limiting mixed strategy equilibrium in symmetric case is the efficient pure strategy equilibrium in the uniform auction

Variations: Vickrey Auction

Payments: Every supplier is paid the opportunity cost of its output; i.e. the rival's rejected offer times its excess capacity, plus P for any remaining output.

Equilibrium Bidding:

For any realization of demand, there exists a unique equilibrium in weakly dominant strategies in which suppliers offer prices at marginal cost.

Comparison with uniform and discriminatory:

- Vickrey auction always results in cost efficiency
- but can result in large payments, and thus be outperformed by the uniform or discriminatory auctions.

Variations: Price-Elastic Demand

Demand function: $D(p; \theta)$ with standard assumptions

- the parameter θ defines a family of demand functions s.t. if $\theta_1 < \theta_2$, $D(p; \theta_1) < D(p; \theta_2)$
- "residual monopoly price": $p_i^r(\theta)=argmax_p(pmin[D(p,\theta)-k_j, k_i]\}$
- effective residual monopoly price: $P_i^r = min\{p_i^r, P\}$

Equilibrium Analysis: extension of Proposition 1

There exists a unique threshold θ* such that equilibrium outcomes are of the low-demand case if θ< θ*, and of the high-demand case otherwise.

The comparison across auction formats is similar:

- plus, allocative efficiency gain in the discriminatory auction.

Demand elasticity improves market performance:

- reduces equilibrium price for each quantity supplied
- makes the low-demand state more likely, i.e. larger θ*

Variations: Price-Elastic Demand

Example: Increasing Demand Elasticity

Assumptions: symmetric suppliers, uniform distribution

$b =$	**0**	**0.025**	**0.050**	**0.075**	**0.100**	**0.125**	**0.150**
ER_d	0.250	0.226	0.203	0.183	0.163	0.146	0.130
ER_u	0.375	0.350	0.327	0.304	0.282	0.260	0.240
ER_u/Er_u	*1.50*	*1.55*	*1.61*	*1.66*	*1.73*	*1.78*	*1.85*

- reduces prices in both auctions
- discriminatory auction performance improves more than uniform in this example

Variations: Symmetric Oligopoly

N firms with $c_i = c$ and $k_i = k$, $= 1,\ldots,N$.

Low-Demand State

- occurs when any N-1 firms can satisfy demand, i.e. θ* = (N-1)k
- highest accepted price equals *c*
- as N > ?, θ* > K

Reducing market concentration:

- reduces incidence of high-demand states (uniform auction)
- in the discriminatory auction, intensifies price competition *within* high-demand states

Symmetric Oligopoly (cont.)

Example: Increasing the Number of Suppliers

Assumptions: Symmetric suppliers, uniform distribution

Table 5

N	**2**	**3**	**4**	**5**	**10**	**100**	**8**
$\mathbf{ER_d}$	0.250	0.167	0.125	0.100	0.050	0.005	0
$\mathbf{ER_u}$	0.375	0.278	0.219	0.180	0.095	0.010	0
$\mathbf{ER_u/ER_d}$	1.50	1.67	1.75	1.80	1.90	1.99	1

- as in previous examples, performance improves faster in discriminatory auction
- in the limit, they are equivalent