MEMORANDUM

No 27/97

CAPACITY INVESTMENT AND COMPETITION IN
DECENTRALISED ELECTRICITY MARKETS

By

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ISSN: 0801-1117

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Department of Economics

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CAPACITY INVESTMENT AND COMPETITION IN DECENTRALISED ELECTRICITY MARKETS

by

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November 1997

Abstract: With particular reference to the recently deregulated and market-based electricity industries in Norway, the UK and elsewhere we analyse oligopoly entry and capacity investment decisions as a non-cooperative game in a decentralised electricity market. We consider a two-stage game similar to Kreps and Scheinkman (1983), with multiple capacity types and uncertain demand, in which capacity decisions are made prior to spot-market, or price competition. Equilibrium outcomes for different pricing mechanisms or regulatory regimes are analysed. We are particularly concerned with the following questions: Will industry capacity be sufficient to ensure adequate supply security? Does imperfect competition in the spot market lead to an inefficient mix of base-load and peak-load technologies? How do different regulatory policies affect the market outcomes?

Acknowledgements: We are grateful to Geir Asheim, John Vickers and seminar participants in Aarhus, Copenhagen, Gothenburg and Stockholm for insightful comments on earlier drafts. Financial support from NFR Norwegian Council for Research is gratefully acknowledged.

1. Introduction

The past decade has been witness to a wave of decentralization and reform in formerly publicly-owned or operated monopoly electricity supply industries. New electricity markets subject to various degrees of regulatory oversight have been created in the UK, Australia, Norway, Sweden, Chile, Argentina, and Alberta, Canada. And in California a major reform and decentralisation of the electricity supply industry is currently underway. A central feature of the liberalisation of the electricity supply industry has been the introduction of a competitive wholesale electricity spot market, or *pool,* coupled with the privatisation and/or decentralisation of longer run capacity investment decisions. A number of studies - both theoretical and empirical - concerned with the competitive performance of electricity spot markets now exist, c.f. von der Fehr and Harbord (1993) (1997a), Green and Newbery (1992), Wolak and Patrick (1996), and Wolfram (1996). However the likely longer term investment performance of such industries has to date received less attention.

Underlying the introduction of the new decentralised regimes has been a general presumption that by subjecting supply and pricing decisions to market forces, more efficient investment decisions were likely to follow. A key characteristic of the monopolistic electricity supply industries under the old regulatory regimes was grossly inefficient levels of installed generation capacity. The pre-reform Norwegian industry for example, suffered from considerable over-investment, and in the UK, under the old Central Electricity Generating Board, investment in excess capacity was coupled with inefficient technology choices and lack of modernisation of plant. In Australia capacity margins of 40% or more were not uncommon in the pre-reform industry. Apart from some fairly general arguments concerning the benefits of competition and market-based transactions however, the belief that decentralisation and the introduction of competition would bring about improved - or even optimal - investment performance has not been subjected to a rigorous analysis taking into account the particular features of electricity supply industries. An important problem for research is therefore to identify the conditions under which decentralised, market-based decisions in electricity supply industries will result in efficient capacity investments. Our purpose in this paper is to begin such an analysis.

In this paper we analyse capacity investment decisions in an oligopolistic industry which supplies a non-storable commodity such as electricity and in which demand (and possibly capacity availability) are uncertain. Least-cost investment requires that an appropriate mix of base-load, cycling and peaking capacity be installed to meet expected demand at minimum cost, taking into account the pattern of short-run demand fluctuations and the cost of not meeting demand in all periods (i.e. rationing costs). Our analysis is aimed at answering the following questions for a decentralised, market-based system: To what extent will supply be secured in peak-demand periods, i.e. how large will total industry capacity be relative to the optimal capacity level? [1] When multiple technologies (base-load, mid-merit, peak-load etc.) are available, what is the industry technology mix? And how will different regulatory policies affect the outcome? In particular our interest focuses on different approaches to regulating - or modelling - generator bidding and pricing behaviour in electricity pools. We consider different three different approaches to the regulation of spot market pricing behaviour - including the complete absence of regulation - and find that investment

[1] It should be noted that we consider production capacity only, and not other constraints on total system output such as insufficient capacity in the transmission and distribution networks. We also disregard all geographical/spatial issues.

incentives depend finely upon the way in which prices are determined, or bidding behaviour regulated.

Some of these questions have been analysed from the point of view of a perfect planner, i.e. a regulator who has complete information and can implement any desired pricing mechanism and industry structure, by a series of authors, including Steiner (1957), Boiteux (1960), Turvey (1968), Crew and Kleindorfer (1971, 1976) and Chao (1983).[2] This literature characterises first-best conditions for technology choice, capacity mix and reliability criteria as well as optimal peak-load pricing. These analyses are important for a number of reasons: in particular, in many countries the electricity industry (as well as other industries with similar characteristics, e.g. gas) is still organised on a non-market basis, and all economic decisions are taken by a central, typically public, authority. For such industries the results from the peak-load-pricing literature are directly applicable. However, the international wave of decentralisation and privatisation in recent years has brought with it a substantial transfer of decision-making authority from central authorities to the market, and investment and generation decisions have been more or less completely decentralised in many countries. In these markets, as noted, the exchange of power is organised through competitive spot (and contract) markets, and long-run capacity decisions are taken by independent generating companies. For such industries the traditional analysis may serve as a useful normative reference point, however as a positive theory it is of limited value.

This paper considers an industry with technological characteristics similar to those assumed in the 'classical' peak-load literature, see e.g. Crew and Kleindorfer (1986).[3] Unlike in the traditional theory however, we assume that investment decisions are taken by independent firms on the basis of expected profit calculations. Investment and supply decisions are modelled as a two-stage non-cooperative game. In the first stage firms make their capacity investment decisions, and in the second stage they compete to supply the spot market. In electricity spot markets a central market organiser is responsible for setting market price and co-ordinating despatch of generating units based on hourly or daily generation supply bids (i.e. 'offer prices') from individual suppliers. Spot market competition is therefore modelled as a non-cooperative game in which firms submit offer prices, a central market organiser (i.e. an auctioneer) determines market price, and the despatch of generating units is in order of increasing offer prices. Investment decisions of firms are analysed for industries having these characteristics, under different assumptions concerning the way in which spot market competition occurs and hence electricity prices determined. In the UK, Australia, and Scandinavia for example, generator bids into the electricity pool are unregulated.[4] In Argentina, by contrast, the pool is regulated, and generators are required to submit (audited) marginal cost based bids.

Under marginal-cost pricing - equivalent to optimal spot pricing in our model - there is a tendency towards under-investment in industry capacity, at all technology levels, whenever firms' capacity choices have a non-negligible effect on market prices. This is for the usual Cournot-type reasons: an increase in investment in any capacity type lowers expected market price in one or more states of the world, and hence lowers the return on existing capacity

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[2] von der Fehr and Harbord (1997b) provides an overview of the literature as part of a comprehensive discussion of capacity investment in electricity industries.

[3] While we do not consider consumer rationing or random availability of capacity, it would appear that our analysis can be extended to include these cases as well.

[4] Although recently the bids of the two major generators in the England and Wales pool were subject to an average price cap.

units. As firms become 'small' this 'external' effect disappears. This result is of some interest since it is often claimed that 'optimal spot pricing' provides incentives or 'signals' for efficient capacity investments - see Schweppe et al (1988) for the classic, and deservedly influential, text on this subject. However this is only true under the assumption that firms ignore the effects of their capacity decisions on spot prices, i.e. behave as 'perfect competitors.' This is of course unrealistic, as is the assumption that generators will become 'small' relative to the size of the market. Hence capacity regulation may be necessary in order to achieve the first best outcome. In the absence of such regulation prices must either be distorted upwards from first-best levels, or there will be under-investment in total capacity.

In contrast, if firms bid according to 'fixed mark-up', or above-cost bidding strategies, we show that either over-investment or under-investment in total capacity may occur. In some cases first best total capacity may be achieved when firms' bids on the peak load technology are sufficiently high.

We then turn to an analysis of investment incentives based on an equilibrium analysis of unregulated bidding behaviour in electricity spot markets, following the analysis in von der Fehr and Harbord (1993). In von der Fehr and Harbord (1993) we modelled spot market competition as a non-discriminatory, multiple-unit, sealed-bid reverse auction and characterised equilibrium outcomes for different levels of the demand/capacity balance.[5] We find here that even though firms have access to the same set of technologies, equilibrium capacity configurations will be asymmetric. Total industry capacity is generally below the first-best level, but higher than under marginal-cost pricing. If prices are allowed to approach consumers' marginal willingness to pay, oligopoly capacity will be close to first-best levels.

The remainder of the paper is organised as follows. In Section 2 we consider the case of a single available technology type. We first describe the basic structure of the model and derive the first-best policy and pricing rule under the assumption that a perfect planner maximises the sum of producers' and consumers' surplus. We then analyse capacity investment decisions under three alternative assumptions concerning how spot prices are set: (i) generators bid their marginal running costs on each unit of capacity; (ii) generators submit regulated 'fixed mark-up' bids; and (iii) generator bids are the equilibrium outcome of the spot market game. In Section 3 we extend the analysis of section 2 to the case of multiple technology types. Section 4 concludes.

2. Industry capacity and supply security

We consider a two-stage game played by oligopolistic firms under a given regulatory regime, i.e. given a particular pool pricing mechanism or pool bidding behaviour. In the first stage n firms simultaneously enter the industry and choose the amount of capacity to install. Then demand is realised and firms simultaneously submit offer prices, or bids at which they are willing to supply power. Units are ranked according to their offer prices, i.e. a supply curve is constructed. Despatch is according to the rank order and market price is determined as the offer price of the marginal unit.

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[5] Modelling competition in pools is a complex matter, and Green and Newbery (1992) have suggested an alternative approach applying the supply function framework of Klemperer and Meyer (1989). See Wolak and Patrick (1996), Newbery (1997) and von der Fehr and Harbord (1997a) for a discussion of the empirical evidence and the relative merits of the alternative approaches.

The assumption that firms enter simultaneously is important in that questions of leader-follower behaviour and strategic entry deterrence cannot be considered. It seems natural to abstract from such questions here, although this limits the scope of the analysis somewhat.

The assumption that demand is realised before prices are set greatly simplifies the analysis. This assumption corresponds to that made in the classical peak-load pricing model and implies that price always clears the market, i.e. there is never rationing of demand. In reality pricing and consumption decisions cannot be made quickly enough to establish 'electrical' equilibrium in all states of the world and consequently rationing will occur in states in which available industry capacity is insufficient to meet total demand. On the other hand, demand tends to vary very systematically so that when bids are made for short time intervals generators will be able to forecast demand with a high degree of accuracy, and the residual uncertainty will be small. This is the case in the Scandinavian pool (Nord Pool), for example, where hourly bids are submitted a day ahead. In the England and Wales pool, on the other hand, where a single price bid is foxed for a whole day, the assumption that demand does not vary within a pricing period is less satisfactory. Nevertheless, the qualitative features of our results do not appear to depend importantly on the assumption that price always clears the market.[6]

Despite the formal similarities between our model and those of Kreps and Scheinkman (1983) and Davidson and Deneckre (1986), there is a fundamental difference in the way market prices are determined. In these 'Betrand-Edgeworth' type models consumers pay the price of the firm from which they are supplied. Typically this implies that consumers pay different prices and hence demand facing lower-pricing firms has to be rationed. However, the pricing mechanism in electricity pools is not Bertrand-like; each unit of capacity receives the same price (system marginal price) and as long as price clears the market demand does not have to be divided between firms by the use of a rationing scheme.

In the absence of strategic entry deterrence or collusion firms will never wish to hold excess capacity. Thus, it makes sense to assume that - contrary to the reality in both the UK and Scandinavian industries - in the spot-market competition stage, output choice is not an issue and firms submit offer prices for their entire capacities.[7]

Note that the above game will always be analysed for a given regulatory regime. Thus implicitly it is assumed that the regulatory regime is not subject to revision and further that the government is fully able to commit to regulatory rules once they are laid down. In practice, investment decisions in electricity industries may - due to their extremely long-term and industry-specific nature - be heavily affected by 'regulatory uncertainty' (Teisberg, 1994). The present analysis abstracts from any incentive problems caused by the regulatory authority's inability to commit to a particular regulatory regime.

Demand and production technology

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[6] Since we do not consider rationing we avoid the problem that (expected) consumer payments may not be sufficient to cover (expected) generation operating and capital costs. Depending on the nature of demand uncertainty and the rationing scheme, in cases when rationing may occur there will typically be a surplus or deficit of revenues over production costs when capacities are at their first best levels. See von der Fehr and Harbord (1997b) for a discussion of the implications of this result for the analysis of market-based industries.

[7] In the UK and Australia generators also submit 'capacity declarations' and 'redeclarations' stating the amount of capacity of each type available to supply power. This has permitted the generators in England and Wales to manipulate the pricing mechanism, which calculates SMP (system marginal price) ex ante on the basis of initial capacity availability declarations. These have been documented in a report by the regulator Offer (1991).

The assumed technological framework is a special case of the classical peak-load pricing model in which the state space is continuous (see e.g. Crew and Kleindorfer, 1986). Random availability of installed capacity is not explicitly considered. Although outages are in practice an important feature of electricity supply industries, the results presented here will also hold in the more general model. In fact it is relatively straightforward to extend our model to allow for uncertain capacity availability (so long as uncertainty is resolved before prices are set), albeit at the cost of complicating the analysis (see Crew and Kleindorfer, 1986, and Chao, 1983, for treatments of outages).

Let the inverse demand function be given by

$$p = \theta w(q)$$

where w(q) is continuously differentiable and decreasing in q and θ is distributed according to the continuously differentiable distribution function F(θ).[8] w(q) may be interpreted as the marginal willingness to pay of consumers when θ = 1. The demand function may then be expressed as

$$q = d(p,\theta) = w^{-1}(\frac{p}{\theta})$$

Unit variable (i.e. marginal) costs of generation v, and unit capital costs c, are assumed constant. At output levels below capacity k, i.e. when q < k, marginal costs of supply equals variable costs v, while marginal costs are infinitely large at output levels exceeding capacity, i.e. when $q \geq k$. Capital is assumed to be perfectly divisible ex ante. We also assume complete information in the sense that firms face common costs and the demand function, the supply technology and the pricing mechanism are assumed common knowledge.

First-best capacity

Before analysing equilibrium capacity investment decisions, we present the first-best solution to the choice of capacity and the prices charged to consumers under the assumption that a planner with complete information maximises a social welfare function. This provides a useful benchmark for discussing alternative regulatory regimes and their performance.

Efficient use of existing capacity requires that each unit of demand which is willing to pay at least the marginal cost of supply get satisfied whenever demand does not exceed the capacity constraint. In this case market price should equal variable unit cost, i.e. p = v whenever d(v,θ) < k. When capacity is insufficient to cover demand at a price equal to variable unit cost, demand must be 'rationed'. Efficient rationing requires that the market price is raised so as reduce demand until it no longer exceeds available capacity; i.e. in 'peak periods', when d(v,θ) > k, p = θw(k).

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8 The assumption that the random parameter enters the (inverse) demand function multiplicatively is purely to simplify the analysis. It appears relatively straightforward, albeit cumbersome, to extend the analysis to more general formulations of demand uncertainty.

We define W to be 'total willingness to pay', i.e. the area under the demand curve, θW(q) = 🖐q θw(q), or w(q) = W☠(q). The first-best capacity may be found by maximising expected 'social welfare', defined as total willingness to pay less total costs, i.e.

$$\int_{\underline{\theta}}^{v/w(k)} \left[\theta W(d(v,\theta)) - vd(v,\theta)\right] dF(\theta) + \int_{v/w(k)}^{\bar{\theta}} \left[\theta W(k) - vk\right] dF(\theta) - ck$$

The first-order condition for the optimal capacity level may be written:

$$Pr(p > v)\left[E(p|p > v) - v\right] = c$$

where Pr(p>v) = 1-F(v/w(k))0 and E(p|p>v) = E(🗎|🗎>v/w(k))w(k). Thus a necessary condition for an optimum is that the (expected) gain from a marginal increase in capacity equals the cost per unit of capacity. The gain from a marginal increase in capacity is given by the (expected) premium of market price over variable unit cost in peak periods. Note that, due to the assumption of constant returns to scale, at the optimum consumer payments are just sufficient to cover costs, i.e. profits are zero.

2.1 Perfectly competitive pool

We being our analysis of equilibrium capacity investments by considering the case in which firms are forced to submit bids on each generating unit equal to marginal operating costs. That is, under this regulatory regime - which approximates the regulatory structure in Argentina - each generator is required to bid at marginal operating cost, and declared costs are subject to audit by the regulatory authorities (see London Economics (1992) for a discussion). As we shall show, under-investment in total industry capacity occurs, essentially for the same reason that Cournot oligopolists do not supply the competitive output - that is, because capacity decision, like output choices, affect price with positive probability. If we increase the number of (symmetric) firms exogenously, then the amount of under-investment decreases as the number of firms becomes large, and disappears in the limit.

Assume price equals marginal cost whenever there is sufficient capacity available and set so as to clear the market otherwise. Given that the aggregate capacity of its competitors are k_{-i}, the profit of firm i which has a capacity of k_i is given by

$$\pi_i = \int_{v/w(k_{-i}+k_i)}^{\bar{\theta}} \left[\theta w(k_{-i} + k_i) - v\right] k_i dF(\theta) - ck_i$$

The first term (on the right hand side) is firm i's net revenues in the event that demand exceeds industry capacity, and price is equal to marginal willingness to pay for capacity increases (in all other events, net revenues are zero). The second term is capacity costs.

At equilibrium,[9] when for all i, $k_i = k/n$, the first-order condition becomes

$$Pr(p > v)\left\{E(p|p > v)\left[1 - \frac{1}{n\varepsilon}\right] - v\right\} = c$$

where $\varepsilon = -w(k)/[kw'(k)] > 0$ is the price elasticity of demand at full capacity utilisation. By comparing this condition with the condition for optimal investment derived above, we find that investment incentives are different for a welfare-maximising planner and a profit maximiser because changes in the price that consumers expect to pay affects profits and welfare in different ways. The external effects from price changes following a change in the output of a particular firm are similar to what one finds in any standard Cournot model; in addition to the firm's own profits, both competitors' profits as well as consumer surplus are affected. Since producers do not take into account the positive effect on consumer surplus from a lower price, the price tends to become to high. On the other hand, the external effect on competitors profits is negative and leads to over-investment ceteris paribus. Since, at the first best capacity level, $\partial W/\partial k = 0$, the former effect outweighs the latter and thus the overall effect leads to under-investment. As n goes to infinity, equilibrium approaches the first-best.

2.2 Imperfectly competitive pool

We assume in this section that each firm submits identical bids $b \geq v$. In general, such bids will not constitute equilibria of the second-stage bidding game (see below). Our purpose in this section is to place some bounds on the capacity investment strategies which could be observed as equilibrium strategies, and to draw out some more general, qualitative properties, and not to characterise investments for equilibrium bidding behaviour. Nevertheless, there are at least two settings in which such bidding behaviour might occur: First, since as noted in the previous subsection, if generators are forced to bid at marginal cost under-investment results, a regulator may deliberately want to distort bids above marginal costs in order to stimulate investment. This could be done by requiring generators to submit (audited) bids that includes a positive profit margin. Second, if firms collude identical bidding strategies may provide a natural focal point for symmetric firms.

Assume then that price equals $b \geq v$ whenever there is sufficient capacity to cover demand, and set so as to clear the market otherwise. The profit function for firm i will now in general depend upon the amount it is called upon to supply in the event that capacity is not fully utilised. We begin by assuming that firms are ranked randomly, so that each firm supplies (in expected terms) 1/nth of the amount of marginal capacity required to meet demand. At the end of this section we discuss the importance of this assumption for the result that investments are increasing in the level of bids b.

We are again looking for a symmetric equilibrium. The profits of firm i, given that the capacity of each of its competitors is k/n, is

[9] See Tirole (1988), chap 5.7, for a discussion of sufficient conditions for an equilibrium to exist and be unique. Tirole also discusses convergence of Cournot equilibrium to the competitive equilibrium when the number of firms goes to infinity.

$$\pi_i = [b - v]\frac{1}{n}\sum_{j=1}^{n} \int_{b/w(\frac{j-1}{n}k)}^{b/w(\frac{n-1}{n}k+k_i)} \min\{d(b,\theta) - \tfrac{j-1}{n}k, k_i\} dF(\theta)$$

$$+ \int_{b/w(\frac{n-1}{n}k+k_i)}^{\bar{\theta}} [\theta w(\tfrac{n-1}{n}k + k_i) - v] k_i dF(\theta) - ck_i$$

The first term is firm i's profits in the event that total industry capacity is not fully utilised. Each element in the sum corresponds to a particular rank of firm i. In the event that firm i is ranked jth, firm i is called upon to supply whenever demand exceeds the capacity of the j-1 firms ranked first, i.e. when $d(b,\theta) > [j-1]k/n$. Firm i produces below full capacity if demand is below the combined capacity of firm i and the j-1 firms ranked before it, i.e. when $d(b,\theta) < k_i+[j-1]k/n$, and produces at full capacity otherwise. The second term corresponds to firm i's profits in the event that total industry capacity is fully utilised.

At the symmetric equilibrium, the first-order condition for profit maximum becomes

$$[b - v]\frac{1}{n}\sum_{j=1}^{n} Pr(\tfrac{j}{n}k < d < k) + Pr(p > b)\left\{E(p|p > b)\left[1 - \frac{1}{n\varepsilon}\right] - v\right\} = c$$

For the case b = v, this condition reduces to the condition when spot market bidding is perfectly competitive, as it should, and we have under-investment. When b > v, for any realisation of demand in which total capacity is not fully utilised, firm i earns an extra [b-v] on its marginal capacity unit in the event that firm i is ranked jth and demand exceeds the capacity of the j first firms. Ceteris paribus this effect tends to increase investment. On the other hand, as b increases (in expected terms) demand is reduced. Consequently, there are two opposing effects and in general we cannot predict whether industry capacity is increasing or decreasing in b. Below we consider two examples in which over-investment may result.

Example (constant elasticity of demand)

Consider first an example in which the elasticity of demand is constant and marginal willingness to pay is uniformly distributed. Let $w(q) = q^{-1/\varepsilon}$ and assume that $\theta \sim U[0,1]$. Then the equilibrium condition becomes

$$[b - v]\frac{b}{n}\sum_{j=1}^{n}\left\{k^{1/\varepsilon} - \left[\tfrac{j}{n}k\right]^{1/\varepsilon}\right\} + \tfrac{1}{2}\left[k^{-1/\varepsilon} - b^2 k^{1/\varepsilon}\right]\left[1 - \frac{1}{n\varepsilon}\right] - \left[1 - bk^{1/\varepsilon}\right]v = c$$

If we let v = 0.1 and c = 0.405, the first-best capacity is k = 1 (for any value of the price elasticity ε). The table below shows how the oligopoly outcome depends on the bid at which generators offer to supply (b), as well as on the number of participating firms.

Table 2.1: Oligopoly industry capacity in per cent of optimal capacity
(v = 0.1, c = 0.405, ε = 1,5)

No of firms	b = v	b = 2v	b = 3v	b = 4v
1	19.1%	19.2%	19.2%	19.2%
2	54.3%	54.8%	54.8%	54.6%
5	80.7%	81.3%	81.5%	81.1%
10	90.2%	91.0%	91.2%	90.8%
25	96.0%	96.9%	97.2%	96.8%
100	99.0%	99.9%	100.2%	99.9%

Industry capacity is increasing in bids when bids are close to marginal cost, and decreasing in bids for higher bid levels. Industry capacity is well below the first best as long as the number of firms is less than 10. On the other hand, industry capacity is increasing in the number of firms, and for sufficiently many firms industry capacity is close to the first-best level. At least in this example, introducing more firms into the industry (e.g. by splitting up existing generators) is a more effective means of improving capacity incentives than to allow firms to increase bids above marginal costs.

Example (inelastic demand)

We next consider an example in which demand is completely price inelastic below a reservation price w; that is, demand equals d whenever price is at or below w and equals zero otherwise.[10] d is a random variable distributed according to the continuously differentiable distribution function G(d). In this example the condition for optimal capacity becomes [w-v][1-G(k)]=c, where the (expected) capacity premium equals the difference between the reservation price and the variable unit cost in the event that demand exceeds capacity. The equilibrium condition is

[10] More accurately, we assume that when price is equal to w consumers are indifferent between consuming and not consuming the good, and hence any level of supply between 0 and d is consistent with equilibrium.

$$[b-v]\frac{1}{n}\sum_{j=1}^{n}\left[G(k)-G(\tfrac{j}{n}k)\right]-[w-b]\frac{k}{n}G'(k)+[w-v][1-G(k)]=c$$

When b = v, we again have under-investment. At the other extreme, when b = w, the equilibrium condition reduces to

$$[w-v]\left[1-\sum_{j=1}^{n}G(\tfrac{j}{n}k)\right]=c$$

Consequently, in the monopoly case (i.e. n=1), capacity investments will be optimal as long as the firm is paid a fixed price equal to the consumer reservation price in all events (note that, since demand is completely inelastic, this outcome is in fact first best). In the oligopoly case (i.e. n>1), setting b = w induces over-investment. Over-investment is greater the more firms there are in the industry.

The two examples illustrate that the exact connection between bids and capacity investments depend on the assumed demand technology. More importantly, they highlight the major insight that there may often be a trade off between optimal spot pricing and optimal capacity investment, and that allowing bids to raise above marginal cost may mitigate under-investment problems. There are two effects:

- The higher are bids, and hence the spot price in the event that demand is below full capacity, the less is the price effect on infra-marginal units of expanding capacity.
- If firms have a positive probability of being despatched with their entire capacity also in cases in which industry capacity is not fully utilised, there is an additional incentive to expand capacity.

If firms are ranked in a fixed order, the second effect disappears as the firm which is ranked last becomes in effect a monopolist with respect to residual demand. On the other hand, unless firms are ranked in a fixed order, the second effect becomes more important the more firms there are in the industry. This explains why in this model we will have over-investment as the number of firms in the industry becomes very large. In particular, from the condition for equilibrium capacity we find that as n goes to infinity, the first element, corresponding to revenue earned on the marginal capacity unit in the event that demand is below industry capacity, remains positive; that is, in the limit, as $n \to \infty$, the equilibrium condition reduces to

$$[b-v]\left[\Pr(d<k)-\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n}Pr(d<\tfrac{j}{n}k)\right]+Pr(d>k)\left[E(p|p>b)-v\right]=c$$

Although the simple model considered in this section illustrates some of the effects involved in determining investment incentives in an imperfectly competitive industry, an obvious objection to the model is the assumption that bids, and hence the spot price in the event that demand is below industry capacity, are independent of capacity investments. In general we would expect bidding behaviour in the pool to be affected by the availability of capacity. We now turn to a model with endogenously determined bids in which this is indeed the case.

2.3 A two-stage duopoly game

We consider a two-stage game, in which firms first choose capacities and then compete in the spot market by choosing bids at which they are willing to supply. For analytical convenience we consider a case in which demand is completely price inelastic below a given reservation price w. In appendix 1, we demonstrate that the results generalise straightforwardly to the case of price elastic demand, albeit at the cost of complicating the analysis somewhat and without adding substantial new insights.

At the price competition stage equilibrium outcomes depend on the installed capacities of the generators. We assume that if there is sufficient capacity to cover all of demand, price is set equal to the bid of the marginal generator. Whenever capacity is fully utilised, price is set so as to clear the market, i.e. firms are paid the consumer reservation price w.

In this section we consider the duopoly case, and leave the analysis of the oligopoly case to the next section. Order firms such that $k_1 \le k_2$ and let $\{p_1,p_2\}$ denote the profile of firms' bidding strategies.[11] We assume that price bids must be chosen below some maximum price P. We will interpret this limit as a cap on bids in the pool (in the absence of such a cap consumer reservation price would provide an upper limit on bids). Such a cap was introduced for the major generators (National Power and PowerGen) in the England and Wales pool recently.[12] The inclusion of P allows us to consider the effects on industry investment from regulatory intervention in the pool.

From the analysis in von der Fehr and Harbord (1993), Propositions 2 and 3, we have the following result which characterises Nash equilibria of the second-stage spot market game:

<u>Proposition 1</u>: Define $b_i = P[d0-k_j]/k_i$, $i,j = 1,2$, $j \neq i$. Equilibria in the pool stage game are

(i) $d0 \le k_1$: $\{v,v\}$

(ii) $k_1 < d0 \le k_2$: $\{p_1,P\}$, $p_1 \le b_1$.0

(iii) $k_2 < d0 \le k_1+k_2$: $\{p_1,P\}$, $\{P,p_2\}$, $p_i \le b_i$,0 $i = 1,2$, and mixed-strategy equilibrium.

(iv) $k_1+k_2 < d0$: Any strategy combination $\{p_1,p_2\}$, $p_i \le P$, is an equilibrium.

When demand does not exceed the capacity of the smallest generator (i.e. $d \le k_1$) there exists a unique equilibrium in which both firms bid at variable unit costs. This case is equivalent to the standard Bertrand model, and firms compete fiercely in prices to be allowed to supply.

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[11] The assumption that a firm must submit one bid for its entire capacity is without loss of generality. Basically the same results would be obtained if one assumed instead that capacities were divided up into smaller units, and firms could make separate bids for each unit.

[12] Even in those countries in which pool bids are in principle completely unregulated, it seems very unlikely that industry regulators would accept spot prices, or bids, at any level. An alternative interpretation of P is therefore the price level at which generators believe regulatory intervention would occur.

When demand exceeds the capacity of the smallest generator (i.e. $d0 > k_1$) there are multiple equilibria. However, all of the pure-strategy equilibria have the same form - one firm bids at the upper limit P, while the other firm bids sufficiently below P to avoid being undercut. The low-bidding firm is despatched with its entire capacity, while the high-bidding firm supplies the residual demand. The intuition for this result is the following. Given that demand is perfectly inelastic, the high-bidding firm, which determines the system marginal price, increases profits by raising its bid. The low-bidding firm, which does not influence the system marginal price, is indifferent between all bids below that of the high-pricing firm. To avoid undercutting, however, the low-bidding firm must bid low enough; the b_i's are the critical values for which undercutting is not profitable at equilibrium.

When demand is below the capacity of the larger generator (i.e. $k_1 < d0$ ✁ k_2) there only exist equilibria in which the larger generator acts as the high-bidding firm: If the larger generator bid below the smaller generator, the smaller generator would not be called upon to supply and therefore would want to undercut the larger generator unless it bid at or below marginal cost. Bidding at marginal cost would never be optimal however, since by raising its bid the larger generator could secure positive profits. When demand exceeds the capacity of the larger generator also (i.e. $d0 > k_2$) there exists equilibria in which either the larger or the smaller generator acts as the high-bidding firm. There also exists a mixed-strategy equilibrium. When demand exceeds industry capacity (i.e. $d > k_1+k_2$) bidding strategies are payoff irrelevant and hence any strategy profile constitute an equilibrium.

Note that when $d > k_1$, the outcome in all equilibria in which firm i bids P and firm j bids some price p_j sufficiently below P are equivalent; that is, system marginal price equals P and revenues are $[P-v][d0-k_i]$ and $[P-v]k_j$, respectively. Pure-strategy equilibrium outcomes are illustrated in figure 2.1.[13]

[13] The type of bidding behaviour predicted by the spot market model has found some empirical support. In their original presentation of the model, von der Fehr and Harbord (1993) also provided evidence from the UK pool which appears to be consistent with the model; in particular, bidding strategies seem to depend on the overall level of demand and in periods of high demand there is evidence of asymmetric high-low bidding outcomes. Wolak and Patrick (1996), in a recent and more detailed study of price determination in the England and Wales electricity industry, also lend support to the von der Fehr and Harbord (1993) spot market model.

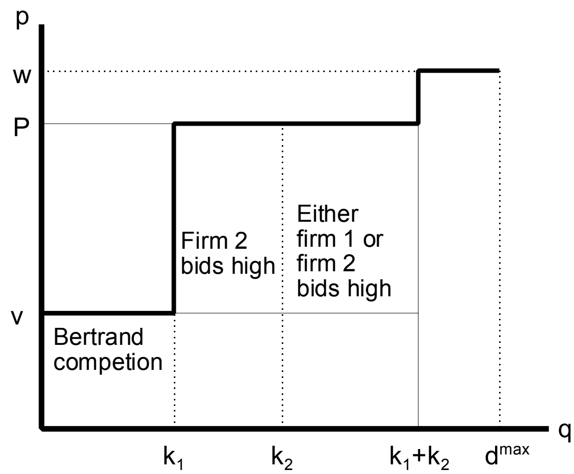

Figure 2.1: Pure-strategy equilibrium pool price

The multiplicity of equilibria in the second state game creates a certain ambiguity for the analysis of the overall game. Since the outcome in the mixed-strategy equilibrium is 'symmetric' (in the sense that revenues are $[P\text{-}v][d0\text{-}k_2]$ to firm 1 and $[P\text{-}v][d0\text{-}k_1]$ to firm 2, see appendix 2), one could argue that the mixed-strategy equilibrium is a natural 'focal point'. On the other hand, from the perspective of the two generators the pure-strategy equilibrium outcomes Pareto dominate the mixed-strategy equilibrium outcome. We therefore focus on pure-strategy equilibria here. As we demonstrate in appendix 2, second-stage outcomes, and hence investment incentives, do not differ much between the alternative equilibria. We could therefore choose one of the two pure-strategy equilibrium outcomes. However, to preserve maximum symmetry ex ante, we assume instead that at the investment stage expected profits equal the average over the two pure-strategy equilibrium outcomes. One may interpret this as the outcome of a 'correlated equilibrium' (Aumann, 1974), in which some (un-modelled) public signal leads firms to co-ordinate on either equilibrium with equal probability.

Then, assuming that in states in which $k_2 < d0 < k_1+k_2$ with probability equal to [illegible] firms play either of the two pure-strategy equilibria, at the investment stage we have the following expected profits (remember that firms are ordered such that $k_1 \leq k_2$):

$$\pi_1 = [P - v]\left\{\int_{k_1}^{k_2} k_1 dG(d) + \int_{k_2}^{k_1+k_2} \left\{½k_1 + ½\left[d - k_2\right]\right\} dG(d)\right\} + \int_{k_1+k_2}^{\bar{d}} [w - v]k_1 dG(d) - ck_1$$

$$\pi_2 = [P - v]\left\{\int_{k_1}^{k_2} [d - k_1] dG(d) + \int_{k_2}^{k_1+k_2} \left\{ \tfrac{1}{2} k_2 + \tfrac{1}{2}[d - k_1]\right\} dG(d)\right\} + \int_{k_1+k_2}^{\bar{d}} [w - v] k_2 dG(d) - ck_2$$

where G(d) is again the (continuously differentiable) distribution function for demand d. Firms earn zero net revenues in the event that demand is below the capacity of firm 1. The first term in each profit expression corresponds to the event that demand is in the range between the capacities of the two generators. Then there is a unique equilibrium in which firm 2 bids high and serves residual demand while firm 1 is despatched with its entire capacity. The second term in the profit expressions corresponds to the event that demand exceeds the capacity of firm 2 but is smaller than industry capacity. Then, with equal probability, either firm bids low and produces at full capacity or bids high and only supplies residual demand. The third term corresponds to the event that demand exceeds available capacity, and therefore both firms produce at full capacity and are paid a price equal to the consumer reservation price.

From these expressions, and under the assumption that $k_1 < k_2$, we get the following first-order conditions for optimal capacities:

$$\frac{\partial \pi_1}{\partial k_1} = [P - v]\left[\tfrac{1}{2} G(k_1 + k_2) + \tfrac{1}{2} G(k_2) - G(k_1) - k_1 G'(k_1)\right] + [w - v]\left[1 - G(k_1 + k_2)\right]$$
$$- [w - P] k_1 G'(k_1 + k_2) - c = 0$$

$$\frac{\partial \pi_2}{\partial k_2} = [P - v]\left[\tfrac{1}{2} G(k_1 + k_2) - \tfrac{1}{2} G(k_2) - \tfrac{1}{2} k_1 G'(k_2)\right] + [w - v]\left[1 - G(k_1 + k_2)\right]$$
$$- [w - P] k_2 G'(k_1 + k_2) - c = 0$$

When P = v, prices equal marginal cost whenever industry capacity is sufficient to cover demand and the first-order conditions corresponds to those derived in section 2.1. above. In this case there is a unique equilibrium in which capacities are symmetric and there is under-investment relative to the first-best outcome.

When P > v, however, there does not exist a symmetric equilibrium: Assume that such an equilibrium exists and, in particular, that the second first-order condition is satisfied. Subtract the second of the two first-order conditions from the first and let $k_1 \to k_2$. This gives $\partial\pi_1/\partial k_1 = -\tfrac{1}{2}[P-v]k_1 G'(k_1)$, which is negative as long as P > v, contradicting the assumption and implying that firm 1 could increase profits by reducing its capacity.

We conclude that equilibrium capacities are asymmetric, and illustrate with an example:

Example (uniformly distributed demand)

Assume that d is uniformly distributed on [0,1]. Then the first-order conditions imply

$$k_1 = \frac{[w-v-c][w-v]}{3[w-v]^2 - \tfrac{1}{2}[P-v]^2}$$

$$k_2 = \frac{[w-v-c]\{w-v+\tfrac{1}{2}[P-v]\}}{3[w-v]^2 - \tfrac{1}{2}[P-v]^2}$$

$$k_1 + k_2 = \frac{[w-v-c]\{2[w-v]+\tfrac{1}{2}[P-v]\}}{3[w-v]^2 - \tfrac{1}{2}[P-v]^2}$$

Clearly $k_1 \leq k_2$. When P = v, we get the symmetric outcome $k_1 = k_2 = [2/5][w-v-c]/[w-v]$. Both capacities are increasing in P. When P = w, $k_1+k_2 = [w-v-c]/[w-v]$, which equals the first-best capacity. Consequently, for P < w, industry capacity is below the first-best level.

In the general case, whether or not over-investment may occur depends on the form of the distribution function G(d). In particular, from the first-order condition corresponding to firm 2's profit-maximisation problem we get

$$[w-v]\big[1-G(k_1+k_2)\big] = c + [w-P]k_2 G'(k_1+k_2) - \tfrac{1}{2}[P-v]\big[G(k_1+k_2) - G(k_2) - k_1 G'(k_2)\big]$$

By the well-known C^1-characterization, if G(d) is concave $G(k_1+k_2) < G(k_2)+k_1G'(k_2)$ and the left hand side of the equation is always greater than or equal to c. Consequently, industry capacity is always below the first-best level when G(d) is concave, that is, if the demand distribution is skewed towards low realisations of demand. If, on the other hand, G(d) is convex, the left hand side of the equation will be smaller than c for P sufficiently close to w, and hence in such cases over-investment results.

2.4 A two-stage oligopoly game

In this section we extend the above model to the case when there are more than two firms in the industry.

Order firms such that $0 = k_0 \leq k_1 \leq k_2 \leq \ldots \leq k_n$, where n is the number of firms. Define $K_i = \Sigma_{s=1}^{i} k_s$ as the aggregate capacity of the i smallest firms and $k_{-i} = \Sigma_{s\neq i} k_s$ as the aggregate capacity of all firms other than i. We then have the following equilibria of the second stage, spot-market game, corresponding to those considered in the previous section:

<u>Proposition 2</u>: Define $b_i0 = P[d0-k_{-i}]/k_i$, i,j = 1,2, j ≠ i. Pure-strategy equilibria in the pool stage game are

(i) $d0 \leq K_{n-1}$: $p_i = v$, i = 1,2,...,n.

(ii) i = 1,2,...,n: $k_{-i} < d0 \leq k_{-[i-1]}$: $\forall j \geq i$: $p_j = p$, $p_s \leq b_s0$, s ≠ j.

(iii) $K_n < d0$: any strategy combination $\{p_1,p_2,\ldots,p_n\}$, $p_i \leq P$, is an equilibrium.

When demand is less than the combined capacity of the n-1 smallest firms (i.e. d 0✂ K_{n-1}) we have the Bertrand outcome as a unique equilibrium outcome. When demand exceeds the combined capacity of the n-1 smallest firms (i.e. d >0 K_{n-1}), there exists multiple equilibria. In particular, when demand is in the range between k_{-i} (the combined capacity of all firms except the ith smallest) and $k_{-[i-1]}$ (the combined capacity of all firms except the i-1th smallest), there exists equilibria in which either of the firms i, i+1,...,n acts as the high-bidding firm (there does not exist equilibria in which any of the smaller firms acts as the high bidder because the residual demand faced by such a firm would be zero). In addition to these pure-strategy equilibria, there are also mixed-strategy equilibria. When demand exceeds industry capacity, any strategy profile constitutes an equilibrium.

Again we assume that only Pareto un-dominated equilibria are played and furthermore that when multiple un-dominated equilibria exists each will be played with the same probability, i.e. firms are equally likely to act as the marginal, or high-bidding firm (as long as this is an equilibrium). The profit of firm i may then be written

$$\pi_i = [P - v]\left\{ \int_{k_{-n}}^{k_{-i}} k_i dG(d) + \sum_{j=1}^{i} \int_{k_{-j}}^{k_{-[j-1]}} \left\{ \tfrac{n-j}{n+1-j} k_i + \tfrac{1}{n+1-j}\left[d - k_{-j}\right]\right\} dG(d) \right\} + \int_{k}^{\bar{d}} [w - v] k_i dG(d) - ck_i$$

where $k_{-0} = \Sigma_i k_i$. When d ✂ k_{-n}, all firms bid at marginal cost and net revenues are zero. Therefore, the first term corresponds to events in which k_{-n} < d0 ✂ k_{-i}; then there only exists equilibria in which i bids low and is despatched with its entire capacity. The second term corresponds to events in which k_{-i} < d0 ✂ k, and consequently there is a positive probability that an equilibrium will be played in which firm i acts as the high-bidding firm. In particular, in the event that k_{-j} < d0 ✂ $k_{-[j-1]}$, there exists equilibria in which firm s, s = j,j+1,...,n, is the high-bidding firm. Consequently, when k_{-j} < d0 ✂ $k_{-[j-1]}$, j ✂ i, with probability 1/[n+1-j], an equilibrium is played in which firm i is marginal and is despatched with only part of its capacity, and with probability [n-j]/[n+1-j] another equilibrium is played in which i is despatched with its entire capacity. The last term corresponds to the event that demand exceeds industry capacity and with probability one firm i supplies its entire capacity.

Assuming that $k_1 < k_2 < ... < k_n$, the first-order conditions become

$$\frac{\partial \pi_1}{\partial k_1} = [P - v]\left\{ G(k_{-1}) - G(k_{-n}) + \tfrac{n-1}{n}\left[G(\sum_{j=1}^{n} k_j) - G(k_{-1}) \right] - k_1 G'(k_{-n}) \right\}$$

$$+ [w - v]\left[1 - G(\sum_{j=1}^{n} k_j) \right] - [w - P] k_1 G'(\sum_{j=1}^{n} k_j) - c = 0$$

$$\frac{\partial \pi_i}{\partial k_i} = [P - v]\left\{G(k_{-i}) - G(k_{-n}) + \sum_{j=1}^{i} \tfrac{n-j}{n+1-j}\Big[G(k_{-[j-1]}) - G(k_{-j})\Big] - k_i G'(k_{-n})\right.$$

$$\left. - \sum_{j=1}^{i-1} \tfrac{1}{[n-j][n+1-j]} k_j G'(k_{-j})\right\} + [w - v]\left[1 - G(\sum_{j=1}^{n} k_j)\right] - [w - P] k_i G'(\sum_{j=1}^{n} k_j) - c = 0,\ i = 2,\ldots,n-1$$

$$\frac{\partial \pi_n}{\partial k_n} = [P - v]\left\{\sum_{j=1}^{n} \tfrac{n-j}{n+1-j}\Big[G(k_{-[j-1]}) - G(k_{-j})\Big] - \sum_{j=1}^{n-1} \tfrac{1}{[n-j][n+1-j]} k_j G'(k_{-j})\right\}$$

$$+ [w - v]\left[1 - G(\sum_{j=1}^{n} k_j)\right] - [w - P] k_n G'(\sum_{j=1}^{n} k_j) - c = 0$$

When P = v, these conditions collapse to those derived in section 2.1. and we get a symmetric equilibrium with under-investment.

When P > v, we can use a similar argument to that in the previous section to show that at equilibrium $k_1 = k_2 = ... = k_{n-1} < k_n$. In particular, subtracting the first-order condition for firm i from the first-order condition for firm i-1, yields

$$\frac{\partial \pi_{i-1}}{\partial k_{i-1}} - \frac{\partial \pi_i}{\partial k_i} = [P - v]\left\{\tfrac{1}{n+1-i}\Big[G(k_{-[i-1]}) - G(k_{-i})\Big] + \big[k_i - k_{i-1}\big] G'(k_{-n}) + \tfrac{1}{[n+1-i][n+2-i]} k_{i-1} G'(k_{-[i-1]})\right\}$$

$$+ [w - P]\big[k_i - k_{i-1}\big] G'(\sum_{j=1}^{n} k_j),\ i = 2,\ldots,n-1$$

$$\frac{\partial \pi_{n-1}}{\partial k_{n-1}} - \frac{\partial \pi_n}{\partial k_n} = [P - v]\left\{G(k_{-[n-1]}) - G(k_{-n}) - k_{n-1}\Big[G'(k_{-n}) - \tfrac{1}{2} G'(k_{-[n-1]})\Big]\right\}$$

$$+ [w - P]\big[k_n - k_{n-1}\big] G'(\sum_{j=1}^{n} k_j)$$

These equations imply that for all i = 1,2,...,n-1, when $\partial\pi_i/\partial k_i = 0$, $\partial\pi_{i-1}/\partial k_{i-1} > 0$ and when $\partial\pi_{i-1}/\partial k_{i-1} = 0$, $\partial\pi_i/\partial k_i < 0$. Consequently, as long $k_{i-1} < k_i$ and k_i is set optimally, firm i-1 would like to increase its capacity. Conversely, when $k_{i-1} < k_i$ and k_{i-1} is set optimally, firm i would like to decrease its capacity. We conclude that at equilibrium we must have $k_1 = k_2 = ... = k_{n-1}$. On the other hand, when k_n is set optimally and k_{n-1} is sufficiently close to k_n, $\partial\pi_{i-1}/\partial k_{i-1} \approx -\tfrac{1}{2}[P-v]k_{n-1}G'(k_{-n}) < 0$, implying that firm n-1 would like to reduce its capacity. Consequently, we must have $k_{n-1} < k_n$ at equilibrium.

We can then rewrite the first-order conditions to give two equations for the two unknowns k_1 and k_n:

$$\frac{\partial \pi_1}{\partial k_1} = [P - v]\left\{\tfrac{n-1}{n} G(k) + \tfrac{1}{n} G(k_{-n}) - \tfrac{n-1}{n} G(k - k_n) - k_1 G'(k - k_n) - \tfrac{n-2}{2n} k_1 G'(k - k_1)\right\}$$
$$+ [w - v][1 - G(k)] - [w - P] k_1 G'(k) - c = 0$$

$$\frac{\partial \pi_n}{\partial k_n} = [P - v]\tfrac{n-1}{n}\left\{G(k) - G(k - k_1) - k_1 G'(k - k_1)\right\}$$
$$+ [w - v][1 - G(k)] - [w - P] k_n G'(k) - c = 0$$

We again turn to a specific example:

Example (uniformly distributed demand)

When d is uniformly distributed on [0,1], the first-order conditions imply

$$k_1 = \frac{[w - v - c][w - v]}{[n + 1][w - v]^2 - \tfrac{1}{2}[P - v]^2}$$
$$k_n = \frac{[w - v - c]\left\{w - v + \tfrac{1}{2}[P - v]\right\}}{[n + 1][w - v]^2 - \tfrac{1}{2}[P - v]^2}$$
$$k = [n - 1]k_1 + k_n = \frac{[w - v - c]\left\{n[w - v] + \tfrac{1}{2}[P - v]\right\}}{[n + 1][w - v]^2 - \tfrac{1}{2}[P - v]^2}$$

When P = v, we get the symmetric outcome $k_1 = k_n$ = [w-v-c]/[n+1][w-v]. Capacities are increasing in P. When P = w, k = [w-v-c]/[w-v], which equals the first-best capacity level. Consequently, for P < w, industry capacity is below the first-best level. Moreover, firm capacities are decreasing in n. Aggregate capacity is, however, increasing in n as long as P < w; in particular, k $\rightarrow$ [w-v-c]/[w-v] as n $\rightarrow \infty$.

More generally, we find from inspection of the first-order conditions that - as in the doupoly case - whether or not over-investment may result depends on the shape of the demand distribution G(d). In general, aggregate capacities will approach the first-best levels as the industry structure becomes sufficiently fragmented. Note that in cases when over-investment occurs, this implies that we may have that aggregate capacity levels are *decreasing* in the number of firms in the industry.

We summarise the discussion in this and the previous section in the following result:

Proposition 3: At equilibrium $k_1 = k_2 = ... = k_{n-1}$ ✃ k_n, and

1. Industry capacity is always below the first-best level when P is sufficiently close to v.
2. If G(d) is concave, there is under-investment for any n and P.
3. If G(d) is convex, there is over-investment when P is sufficiently close to w.

3. Technology choice and cost efficiency

When output is non-storable and demand varies least-cost investment requires that an appropriate mix of technologies - which differ with respect to the relative proportions of fixed and variable costs - be installed to meet expected demand at minimum costs. We now turn to the analysis of firms' capacity choices when there are multiple technologies available.

Unit variable costs and unit capital costs are assumed constant for each type of technology. A production technology is defined as a pair {v,c} of unit variable and unit capital costs. It is assume that there are T available technologies $\{v_t,c_t\}$, t = 1,2,...,T, ordered such that $v_t < v_{t+1}$. We assume that all technologies are 'efficient', i.e. are used at equilibrium (Crew and Kleindorfer, 1986). It may be noted here that a necessary though not sufficient condition for efficiency is that if $v_t < v_s$ then $c_t > c_s$, all s,t.

Let k_i^t be firm i's installed capacity of technology t, i = 1,2,...,n, and k^t industry capacity of technology t, t = 1,2,...,T.

Efficient capacity utilisation implies that technologies with low variable unit costs are despatched to meet demand first. Optimal pricing requires that all consumers willing to pay the variable unit cost of the marginal technology be served. Define the following technology-specific, net-of-variable-cost prices, p_t:

$$p_t = \sum_{\tau=t}^{T-1} \left\{ \int_{v_\tau / w(K^\tau)}^{v_{\tau+1} / w(K^\tau)} \left[\theta w(K^\tau) - v_t\right] dF(\theta) + \int_{v_{\tau+1} / w(K^\tau)}^{v_{\tau+1} / w(K^{\tau+1})} \left[v_\tau - v_t\right] dF(\theta) \right\} + \int_{v_T / w(K^T)}^{\bar{\theta}} \left[\theta w(K^T) - v_t\right] dF(\theta)$$

In the event that technology t is marginal, and the installed capacity of this technology is not fully utilised, price is set equal to the variable cost of technology t, v_t (this case corresponds to the second element in the summation). When demand is constrained by K^t (the combined capacities of technologies τ = 1,2,...,t) and t < T, the market-clearing price is in the range $[v_t,v_{t+1}]$ (this case corresponds to the first element in the sum). Finally, when demand is constrained by industry capacity the market clearing price is above the marginal cost of the peak technology, v_T (this case corresponds to the last term in the expression).

Given optimal pricing, the first-order conditions for optimal capacities are $p_t = c_t$, t=1,2,...,T-1, which may alternatively be written:

$$Pr(d = K^t)\left[E(p|d = K^t) - v_t\right] + Pr(d > K^t)\left[v_{t+1} - v_t\right] = c_t - c_{t+1},\ t = 1,2,...,T-1,$$
$$Pr(d = K^T)\left[E(p|d = K^T) - v_T\right] = c_T$$

where $\Pr(d=K^t) = F(v_{t+1}/w(K^t))-F(v_t/w(K^t))$, 0t = 1,2,...,T-1, $\Pr(d=K^T) = 1-F(v_T/w(K^T)$, $\Pr(d>K^t) = 1-F(v_t/w(K^t))$, $E(p|d=K^t) = E(\text{🖹}|v_t/w(K^t)<\text{🖹}<v_{t+1}/w(K^t))w(K^t)$, 0t = 1,2,...,T-1,0 and $E(p|d=K^T) = E(\text{🖹}|v_T/w(K^T)<\text{🖹})w(K^T)$.

We recognise the condition for optimal industry capacity discussed in the previous section. The other conditions determine optimal levels of each technology type. In particular, the gain from replacing a unit of capacity t+1 by a unit of capacity t is partly to increase output in states in which K^t constrains output, and partly to reduce costs in states in which technology t is being used. At optimum this gain equals the difference in capacity costs between the technologies.

In the limiting case, when demand is completely price inelastic below some reservation price w, the prices p_t reduce to

$$p_t = \sum_{\tau=t}^{T-1} \int_{K^\tau}^{K^{\tau+1}} \left[v_{\tau+1} - v_t\right] dG(d) \; + \int_{K^T}^{\bar{d}} \left[w - v_t\right] dG(d)$$

where G(d) is the distribution function for demand, and the equilibrium conditions become:

$$\left[1 - G(K^t)\right]\left[v_{t+1} - v_t\right] = c_t - c_{t+1},\; t = 1,2,...,T-1,$$
$$\left[1 - G(K^T)\right]\left[w - v_T\right] = c_T$$

In this case, it is easy to see that the efficient technology frontier is downward-sloping and convex in {v,c} -space. In particular, to ensure that $k^t > 0$, implying $1-G(K^t) < 1-G(K^{t+1})$, we must have

$$1 > -\frac{c_t - c_{t-1}}{v_t - v_{t-1}} > -\frac{c_{t+1} - c_t}{v_{t+1} - v_t} > \frac{c_T}{w - v_T}$$

For analytical convenience (but without really restricting the generality of our results), for the rest of the paper we restrict attention to the case in which demand is completely price inelastic.

3.1 Perfectly competitive pool

As in section 2, we consider first the case when, on each set, firms are forced to submit bids equal to marginal operating cost.

We assume that there are n firms in the industry, which (by a symmetry assumption) all invest in the whole range of technologies. Given that price is equal to the marginal (operating) cost of the marginal technology in each state of the world in which the overall capacity constraint do not bind, the profit of firm i, i = 1,2,...,n, given the capacity choices of all other firms, is given by:

$$\pi_i = \sum_{t=1}^{T} \left[p_t - c_t \right] k_i^t$$

Assuming interior solutions, we may use first-order conditions to examine capacity investment decision. The first-order conditions for firm i may be written

$$p_t + \sum_{\tau=1}^{T} \frac{\partial p_\tau}{\partial k_i^t} k_i^\tau = c_t, t = 1,2,\ldots,T.$$

In section 2, we noted that investment incentives are different for a welfare maximizer and a profit maximizer because the latter do not take into account the external effects on consumer welfare and competitor profits. Here we identify another reason why investment incentives differ; it is only the profit maximizer who is concerned with how investment in one particular technology affects the profits earned on others. The second term on the left-hand side in the above first-order conditions captures the 'external' effect of an increase in firm j's investment in technology t on the revenues earned on other technologies. Consider for example the equation corresponding to the peak-load technology: Roughly speaking, with 'probability' $G'(K^T)$, a (marginal) increment in firm i's investment in technology T results in a decrease in price from w to v_T, and thus reduces the revenue received by firm i on all capacity types in this event. This effect is negative and thus, ceteris paribus, leads to under-investment.

From the first-order conditions, we get

$$\left\{ 1 - \left[1 + \frac{\eta^t}{n} \right] G(K^t) \right\} \left[v_{t+1} - v_t \right] = c_t - c_{t+1},\ t = 1,2,\ldots,T-1,$$

$$\left\{ 1 - \left[1 + \frac{\eta^T}{n} \right] G(K^T) \right\} \left[w - v_T \right] = c_T$$

where $\eta^t = G'(K^t)K^t/G(K^t) > 0$. Letting K^t0* denote first-best levels, we consequently have

$$\left[1 + \frac{\eta^t}{n} \right] G(K^t) = G(K^t*),\ t = 1,2,\ldots,T.$$

That is, for all t=1,2,...,T, the aggregate capacity of technology t and all technologies less flexible than t, is below the first-best level (i.e. $K^t < K^t*$). We conclude that in oligopoly welfare will improve if the capacity of any technology is increased. Note that although this implies that there is under-investment in at least some technologies relative to first best, it does not follow that there is necessarily under-investment in all technologies (there will always be under-investment in base-load capacity, however, since the above condition for t=1 implies $k^1 < k^1*$). Note that as the number of firms in the industry increases, under-investment is reduced and in the limit equilibrium capacities equal first-best levels.

Example (uniformly distributed demand)

When demand is distributed uniformly, the reduction in capacities is proportional. In particular, the above equations may be written:

$$K^t = \frac{n}{n+1} K^t*,\ t = 1,2,...,T$$

implying

$$k^t = \frac{n}{n+1} k^t*,\ t = 1,2,...,T$$

Consequently, at equilibrium capacities of every technology is reduced proportionally relative to first best.

Example (exponentially distributed demand)

When demand is exponentially distributed, i.e. $G(K) = 1\text{-}e^{-K}$, the absolute, and consequently the relative, difference between first-best and equilibrium capacities is smaller for less flexible technologies (i.e. technologies with low variable unit costs and high fixed unit costs):

$$e^{K^t*\text{-}K^t} = \frac{1}{1-{K^t}/{n}},\ t = 1,2,\ldots,T$$

3.2 A two-stage duopoly game

We now extend the duopoly game considered in section 2.3, in which firms set prices freely in the pool stage game, to the case in which firms may invest in a range of different technologies.

We find that equilibrium bidding behaviour leads to a very natural generalisation of the spot-price result in the single-technology model. When demand can be covered by the capacity of the T-1 lowest-operating-cost technologies, we get a ‘generalised Bertrand’ outcome; in particular, when technology t is marginal bids never exceed the operating cost of technology t+1. When demand is so high that the most flexible technology (technology T) is marginal, bids are constrained by the limit on admissible bids (P) only.

We also find that in many events equilibrium bidding strategies are asymmetric. The asymmetry of bidding behaviour in the pool results in asymmetric payoff functions for the two firms and hence to asymmetric investment incentives. As a result, capacities of the two firms will generally be different. In particular, depending on the underlying demand distribution, equilibrium investment behaviour may involve technology specialisation.

Fix t and order firms such that $k_1^t \leq k_2^t$. Let $\{p_1^t, p_2^t\}$ denote firms' bidding strategies with respect to their technology t capacities. Then equilibrium bidding behaviour in the pool may be characterised by the following result:[14]

Proposition 4: The following set of strategies constitute equilibria in the pool stage game:

(i) $K^{t-1} < d0 \leq K^{t-1}+k_1^t$: $p_i^{☺} = v_\tau$, all i,τ

(ii) $K^{t-1}+k_1^t < d0 \leq K^{t-1}+k_2^t$: $p_1^t = v_t$, $p_2^t = v_{t+1}$; $p_i^{☺} = v_\tau$, all $i,\tau \neq t$

(iii) $K^{t-1}+k_2^t < d0 \leq K^t$: $p_i^t = v_t$, $p_j^t = v_{t+1}$, $i,j = 1,2$, $i \neq j$; $p_i^{☺} = v_\tau$, all $i,\tau \neq t$

where we have defined $v_{T+1} = P$.

Consider the event that demand exceeds the total installed capacity of technologies 1,2,...,t-1 but may be covered if the technology t capacity is used also (i.e. $K^{t-1} < d0 \leq K^t$). By 't-residual demand' we shall here mean the amount of demand which cannot be covered by the aggregate capacity of the installed capacity of technologies 1,2,...,t-1 (i.e. $d-K^{t-1}$). When t-residual demand is smaller than the technology t capacity of firm 1 (case (i) in the proposition) both firms bid in their technology t capacity at marginal operating costs. On the other hand, if t-residual demand exceeds the technology t capacity of firm 1, the equilibrium outcome is asymmetric. When t-residual demand is below the technology t capacity of firm 2 (case (ii)) the equilibrium outcome involves firm 2 bidding its technology t capacity at the marginal operating cost of technology t+1 (firm 2 cannot bid higher because then firm 1 would undercut with the bid on its technology t+1 capacity), while firm 1 bids at marginal cost to avoid firm 2 undercutting it. Lastly, when t-residual demand exceeds the technology t capacity of firm 2 also (case (iii)), there exists asymmetric equilibria in which either firm bids high.

To sum up: When technology t is marginal, the spot price equals the marginal operating cost of technology t in the event that t-residual demand is less than the technology t capacity of firm 1 and equals the marginal operating cost of technology t+1 otherwise. The outcomes are illustrated in Figure 3.1 for an example with two technologies and with $k_1^1 \leq k_2^1$ and $k_1^2 \leq k_2^2$.

Remark: In case (i), two conditions must be satisfied at equilibrium: 1) Both firms bid in their technology t capacity at marginal cost, and 2) capacities of technologies 1,2,...,t-1 are bid in at lower prices (and capacities of technologies t+1,...,T are bid in at higher prices) than the operating cost of technology t. In cases (ii) and (iii), three conditions must be satisfied for an equilibrium to exists: 1) The high-bidding firm, firm i, must bid in its technology t capacity at a price equal to the variable cost of the t+1 technology; 2) the other firm, firm j, must bid in its technology t+1 capacity at marginal cost (to avoid the high-bidding firm increasing its bid); 3) firm j must also bid its technology 1,2,..., and t capacities sufficiently close to variable cost (to avoid the high-bidding firm undercutting it). Consequently, there exists a continuum of payoff-equivalent equilibria to those presented in the proposition. There also exist mixed-strategy equilibria.

[14] It is assumed that when two different sets are bid in at the same price, the set with the lower marginal cost is ranked first.

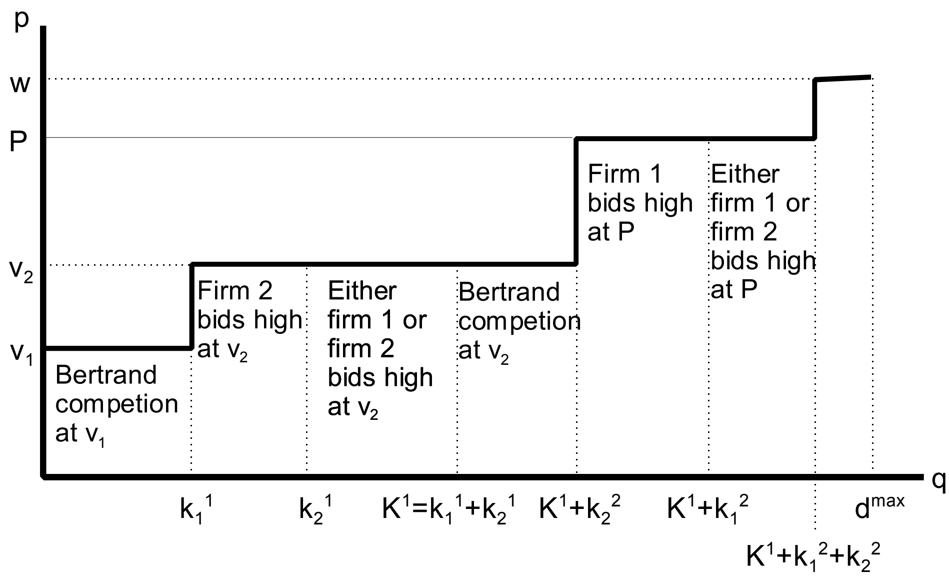

Figure 3.1: Pure-strategy equilibrium pool price in multi-technology duopoly

Assuming that pure-strategy equilibrium is played and ordering firms such that $k_1^T \geq k_2^T$, the first-order conditions for the technology T capacity choices become (see appendix 3 for the derivation of the below results)

$$\frac{\partial \pi_1}{\partial k_1^T} = \left[P - v_T\right]\left[\tfrac{1}{2}G(K^T) + \tfrac{1}{2}G(K^{T-1} + k_2^T) - G(K^{T-1} + k_1^T) - K_1^T G'(K^{T-1} + k_1^T)\right]$$
$$+\left[w - v_T\right]\left[1 - G(K^T)\right] - \left[w - P\right]K_1^T G'(K^T) - c_T = 0$$

$$\frac{\partial \pi_2}{\partial k_2^T} = \left[P - v_T\right]\left[\tfrac{1}{2}G(K^T) - \tfrac{1}{2}G(K^{T-1} + k_2^T) - \tfrac{1}{2}k_1^T G'(K^{T-1} + k_2^T)\right]$$
$$+\left[w - v_T\right]\left[1 - G(K^T)\right] - \left[w - P\right]K_2^T G'(K^T) - c_T = 0$$

These expressions correspond directly to those derived in section 2.3 for the single-technology case. Consequently, we conclude that we cannot have a symmetric equilibrium (as long as $P > v_T$), there is under-investment for P sufficiently close to v_T, there is under-investment for all P if G(d) is concave, while there is over-investment for P sufficiently close to w if G(d) is convex.

We next consider circumstances in which one firm has a larger capacity for each individual technology type; in particular, $k_1^t > k_2^t$, all t. In this case the first-order conditions for technology T-1 reduce to

$$\frac{\partial \pi_1}{\partial k_1^{T-1}} = \left[v_T - v_{T-1}\right]\left[1 - \tfrac{1}{2}G(K^{T-1}) + \tfrac{1}{2}G(K^{T-2}+k_2^{T-1}) - G(K^{T-2}+k_1^{T-1}) - K_1^{T-1}G'(K^{T-2}+k_1^{T-1})\right]$$
$$+\left[P - v_T\right]\left[\tfrac{1}{2}G(K^T) + \tfrac{1}{2}G(K^{T-1}+k_2^T) - G(K^{T-1}+k_1^T) - K_1^T G'(K^{T-1}+k_1^T) + \tfrac{1}{2}k_1^T G'(K^{T-1}+k_2^T)\right]$$
$$+\left[w - v_T\right]\left[1 - G(K^T)\right] - \left[w - P\right]K_1^T G'(K^T) - c_{T-1} = 0$$

$$\frac{\partial \pi_2}{\partial k_2^{T-1}} = \left[v_T - v_{T-1}\right]\left[1 - \tfrac{1}{2}G(K^{T-1}) - \tfrac{1}{2}G(K^{T-2}+k_2^{T-1}) - \tfrac{1}{2}k_1^{T-1}G'(K^{T-2}+k_2^{T-1})\right]$$
$$+\left[P - v_T\right]\left[\tfrac{1}{2}G(K^T) - \tfrac{1}{2}G(K^{T-1}+k_2^T) - \tfrac{1}{2}k_1^T G'(K^{T-1}+k_2^T) - K_2^{T-1}G'(K^{T-1}+k_1^T)\right]$$
$$+\left[w - v_T\right]\left[1 - G(K^T)\right] - \left[w - P\right]K_2^T G'(K^T) - c_{T-1} = 0$$

Subtracting the first-order condition for firm 2 from the first-order condition for firm 1, and using the fact that technology T capacities are set optimally, we get:

$$\frac{\partial \pi_1}{\partial k_1^{T-1}} - \frac{\partial \pi_2}{\partial k_2^{T-1}} = \left[v_T - v_{T-1}\right]\Big\{G(K^{T-2}+k_2^{T-1}) - G(K^{T-2}+k_1^{T-1}) + \tfrac{1}{2}k_1^{T-1}G'(K^{T-2}+k_2^{T-1})$$
$$- K_1^{T-1}G'(K^{T-2}+k_1^{T-1})\Big\} + \left[P - v_T\right]\left[\tfrac{1}{2}k_1^T G'(K^{T-1}+k_2^T) + K_2^{T-1}G'(K^{T-1}+k_1^T)\right]$$

If $P = v_T$, the right-hand side of this expression is negative when k_1^{T-1} is sufficiently close to k_2^{T-1}. On the other hand, if P is large, the expression is always positive, contradicting the assumption that $k_1^t > k_2^t$, all t.

There are two reasons why the firm with the smaller technology T capacity has a stronger incentive to invest in capacities of less flexible technologies when $P >> v_T$:

- Firstly, increasing the capacity of less flexible technologies moves upward the point at which the spot market outcome shifts from firm 1 being despatched with the total of its capacity to both firms having an equal chance of being thus despatched. Firm 2, which by its choice of technology T capacity determines where this shift occurs, takes this effect into account when choosing its technology T capacity. For firm 1 there is no corresponding offsetting effect on its technology T capacity and it consequently obtains a gain at the margin from increasing its technology T-1 capacity (the effect is captured by the term $[P-v_T]\tfrac{1}{2}k_1^T G'(K^{T-1}+k_2^T)$ in the above expression).
- Secondly, increasing the capacity of less flexible technologies moves upward the point at which market price shifts from v_T to P. Firm 1, which by its choice of technology T capacity determines where this shift occurs, takes this effect into account when choosing its technology T capacity. Firm 2 on the other hand, does not and is correspondingly hurt

when increasing its technology T-1 capacity (the effect is captured by the term $[P-v_T]K_2^{T-1}G$☠$(K^{T-1}+k_1^T)$.

We conclude that when bids on the peak-load technology are raised sufficiently above marginal costs, there is technology specialisation: That is, the firm which invests more in the peak-load technology invests less in other technologies, and vice versa. In particular, if there are only two technologies available (i.e. T=2) and $k_1^T < k_2^T$, we have $k_1^{T-1} > k_2^{T-1}$. We summarise the results in the following proposition:

Proposition 5: As long as $P > v_T$, there does not exist a symmetric equilibrium so that $k_1^t = k_2^t$, all t. Furthermore:

1. When P is close to v_T, the firm that invest less in the peak-load technology may invests less in other technologies also; in particular, if T = 2, we may have $k_1^T < k_2^T$ and $k_1^{T-1} < k_2^{T-1}$.

2. When P is sufficiently above v_T, however, there is technology specialisation; in particular, if T = 2 and $k_1^T < k_2^T$, $k_1^{T-1} > k_2^{T-1}$.

4. CONCLUSION

In this paper we have been concerned with investment incentives in industries which supply a non-storable commodity and demand is variable. We have uncovered a number of effects that tend to distort investment incentives.

1. *The consumer price effect:* As in any oligopoly model with imperfect price discrimination, profit-maximising sellers will not take into account the fact that by restricting output, and thus increasing price, (expected) consumer welfare is affected. On the other hand, an increase in price benefits competitors positively. In most reasonable cases, the overall externality is negative and thus, ceteris paribus, leads to under-investment.
2. *The multiple technology firms effect:* When the price paid to a producer depends on his own bids, the amount invested in any particular technology will affect the average price received on sets of other technologies. In particular, when bids on individual sets are increasing in the variable costs of sets, by increasing the capacity of a technology t set, the average price paid to sets of more capital-intensive technologies than t (i.e. technologies 1,2,...,t-1) will be lowered since higher-priced sets of less capital-intensive technologies than t (i.e. technologies t+1,...,T) will be despatched less often, and, furthermore, full capacity utilisation occurs less frequently. This effect, taken in isolation, leads to under-investment.
3. *The non-competitive spot prices effect:* When payment to the marginal operating set exceeds the variable cost on that set, e.g. because bids are distorted upwards from variable costs, there is a super-optimal incentive to invest.

Since these effects work in opposite directions, and, depending on the parameters of the model, each effect may dominate the others, it is not possible to make general statements as to whether over-investment, or under-investment, will occur. However, we have identified two important special cases for which we can provide clear-cut results: When spot-market bids reflect marginal costs, either because of strong price competition or because bids are regulated, the third of the above effects is not present and thus under-investment in total industry capacity, as well as in base-load capacity, will result. On the other hand, when bidding in the pool is unrestricted, demand is sufficiently price inelastic and the demand distribution is skewed towards high realisations of demand, over-investment in total industry capacity results. We note also that the more collusive is price competition in the spot market, the more likely it is that over-investment will occur (c.f. the change from regulated to collusive industry in the UK which has lead to high levels of investment).

In general, aggregate capacities will approach first-best levels as the industry structure becomes sufficiently fragmented. Because of this result, in cases in which the non-competitive bids effect is sufficiently strong, and thus there is over-investment, we may have that aggregate capacity levels are *decreasing* in the number of firms in the industry.

Note that the tendency to over-invest that we have found occurs even though we have assumed that entry by different firms happen simultaneously, something which rules out pre-emptive capacity building. Furthermore, capacity is totally divisible ex ante. If capacity is lumpy, firms may be forced to build larger capacities than what they would like to declare in the price-competition stage. In particular, oligopolistic collusion may lead to restrictions on output. However, if such behaviour results in excess profits in the short run, these may be competed away by capacity building, with the inefficient result of strengthening the tendency to over-capacity in the industry.

Investment in electricity production is both highly industry specific and very long term (partly because lead times are often considerable). The danger that a regulator in the future may tighten prices once investment has been sunk can be a deterrent to efficient behaviour. Note that the mix as well as the level of investment could be affected by regulatory uncertainty, which are greatest for plants that are capital intensive and has long lead times (i.e. nuclear). As far as the technology mix is concerned, this effect would counter any tendency to too few peak-load plants found in the above model.

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APPENDIX 1: Duopoly model with price elastic demand

For states in which $d(v,\theta) > k_j$, define p_i^m to be the price that maximises firm i's revenues relative to residual demand, $d(p,\theta)-k_j$ (taking into account that output is constrained by firm i's capacity); that is,

$$p_i^m = \arg\max_p \left\{ \left[p - v \right] \left\{ \min\{k_i, d(p,\theta) - k_j\} \right\} \right\},\ i,j = 1,2;\ i \neq j.$$

p_i^m will be increasing in demand (θ) and decreasing in the competitor's installed capacity (k_j); in particular, p_1^m ✄ p_2^m whenever k_1 ✄ k_2.

Furthermore, let $P_i = \min\{p_i^m, P\}$. Then we have the following result:

<u>Proposition A.1</u>: Define $b_i0 = P_i[d(P_i,\theta)-k_j]/k_i$, $i,j = 1,2$, $j \neq i$. Then equilibria in the pool-stage game are

(i) $d(v,\theta)$ ✄ k_1: $\{v,v\}$

(ii) $k_1 < d(v,\theta)$ ✄ k_2: $\{p_1,P_2\}$, p_1 ✄ b_10

(iii) $k_2 < d(v,\theta)$: $\{p_1,P_2\}$, $\{P_1,p_2\}$, p_i ✄ b_i0, $i=1,2$, and mixed-strategy equilibrium.

Consequently, P_i plays the same role here as P does in the case of inelastic demand.

When demand is not constrained by capacity at equilibrium, and the monopolistic price p_i^m ✄ P, p_i^m can be found from the first-order condition for the above maximisation problem. The first-order condition may be written

$$p\left[1 - \frac{1}{\varepsilon}\right] = v,\quad \varepsilon = -\frac{\partial d(p,\theta)}{\partial p}\frac{p}{d(p,\theta) - k_j}$$

Clearly, the price p that solves this equation is increasing in the elasticity of demand, ε. In particular, p can be made arbitrarily large by choosing a demand function for which ε is sufficiently close to 1, i.e. for $\partial d(p,\theta)/\partial p$ small enough. Consequently, this more general case collapses to the case considered in section 2.3. as demand becomes completely price inelastic, i.e. when $\partial d(p,\theta)/\partial p = 0$ for all p and θ.

APPENDIX 2: Equilibria in the single-technology duopoly model

In section 2.3 we demonstrated that in the event that there is sufficient capacity available to serve all of demand, but none of the firms have sufficient capacity to cover demand alone, there exists a multiplicity of equilibria in the second-stage game. In particular, there exists two asymmetric pure-strategy equilibrium outcomes in which either of the two firms bid high, and in addition there exists a mixed-strategy equilibrium. In this appendix we sketch equilibria of the overall game in which - with probability equal to one - either of the three second-stage equilibria are played.

Small firm bids high

Assume that it is expected that when a second-stage equilibrium in which firm 1 bids above firm 2 exists, this equilibrium will be played. We consider equilibria of the overall game in which the capacity of firm 1 does not exceed that of firm 2, i.e. $k_1 \leq k_2$.

At the investment stage expected profits are

$$\pi_1 = [P - v]\left\{\int_{k_1}^{k_2} k_1 dG(d) + \int_{k_2}^{k_1+k_2} [d - k_2] dG(d)\right\} + [w - v]\int_{k_1+k_2}^{\bar{d}} k_1 dG(d) - ck_1$$

$$\pi_2 = [P - v]\left\{\int_{k_1}^{k_2} [d - k_1] dG(d) + \int_{k_2}^{k_1+k_2} k_2 dG(d)\right\} + [w - v]\int_{k_1+k_2}^{\bar{d}} k_2 dG(d) - ck_2$$

From these expressions we get the following first-order conditions for optimal capacities:

$$\frac{\partial \pi_1}{\partial k_1} = [P - v][G(k_2) - G(k_1) - k_1 G'(k_1)] + [w - v][1 - G(k_1 + k_2)]$$
$$- [w - P] k_1 G'(k_1 + k_2) - c = 0$$

$$\frac{\partial \pi_2}{\partial k_2} = [P - v][G(k_1 + k_2) - G(k_2) - k_1 G'(k_2)] + [w - v][1 - G(k_1 + k_2)]$$
$$- [w - P] k_2 G'(k_1 + k_2) - c = 0$$

There does not exist a symmetric equilibrium as long as $P > v$. To see this, assume for contradiction that such an equilibrium exists and, in particular, that the first-order condition for firm 1 is satisfied. Subtract the first of the two first-order conditions from the second. Then as $k_2 \rightarrow k_1$, $\partial\pi_2/\partial k_2 \rightarrow [P-v][G(k_1+k_2)-G(k_2)] > 0$, contradicting the assumption and implying that firm 2 could increase profits by expanding its capacity.

From the first-order condition for firm 2 it follows that industry capacity is always below the first-best level when G(d) is concave. On the other hand, if G(d) is convex the left hand side of the equation will be smaller than c for P sufficiently close to w, and hence in such cases over-investment results.

Large firm always bids high

Assume next that it is expected that when a second-stage equilibrium in which firm 2 bids above firm 1 exists, this equilibrium will be played. Again we consider overall equilibria of the game in which the capacity of firm 1 does not exceed that of firm 2, i.e. $k_1 \leq k_2$.

At the investment stage expected profits are

$$\pi_1 = [P - v] \int_{k_1}^{k_1+k_2} k_1 dG(d) + [w - v] \int_{k_1+k_2}^{\bar{d}} k_1 dG(d) - ck_1$$

$$\pi_2 = [P - v] \int_{k_1}^{k_1+k_2} [d - k_1] dG(d) + [w - v] \int_{k_1+k_2}^{\bar{d}} k_2 dG(d) - ck_2$$

From these expressions we get the following first-order conditions for optimal capacities:

$$\frac{\partial \pi_1}{\partial k_1} = [P - v][G(k_1 + k_2) - G(k_1) - k_1 G'(k_1)] + [w - v][1 - G(k_1 + k_2)] - [w - P] k_1 G'(k_1 + k_2) - c = 0$$

$$\frac{\partial \pi_2}{\partial k_2} = [w - v][1 - G(k_1 + k_2)] - [w - P] k_2 G'(k_1 + k_2) - c = 0$$

Assume that the first-order condition for firm 2 is satisfied and use this to simplify the first-order condition for firm 1. Then as $k_1 \to k_2$, $\partial\pi_1/\partial k_1 \to [P-v][G(k_1+k_2)-G(k_1)- k_1G'(k_2)]$. The sign of the last expression depends on the shape of G(d). If G(d) is concave, $\partial\pi_1/\partial k_1$ is negative for k_1 sufficiently close to k_2, implying that firm 1 could increase profits by reducing its capacity. Hence in this case equilibrium must involve asymmetric capacity choices. On the other hand, if G(d) is convex, $\partial\pi_1/\partial k_1$ is positive for all $k_1 \leq k_2$, implying that an equilibrium in which the capacity of the low-bidding firm is less than or equal to that of the high-bidding firm does not exist. If G(d) is linear (i.e. demand is uniformly distributed) a symmetric equilibrium exists.

Note that unlike in the equilibrium considered above, here there can never be over-investment. As seen from the first-order condition for firm 2, there is under-investment as long as $P < w$ and, as P approach w, industry capacity reaches the first-best level.

Mixed-strategy equilibrium

In the second-stage, mixed-strategy equilibrium, firm i's bid, p_i, is distributed on [v,P] according to some distribution function $H_i(p)$. Clearly, the highest admissible bid (P) must be in the support of firms' strategies (since otherwise, given that a firm bids the highest price in the support of the strategies, it could increase profits by raising its bid). When bidding P, firm j earns [P-v][d-k_i] (since with probability 1 the competitor bids lower). For firm j to be indifferent between bidding P and bidding some other price less than P, we must have

$$H_i(p)\left[p-v\right]\left[d-k_i\right]+\int_p^P\left[\rho-v\right]k_j dH_i(\rho) \ = \ \left[P-v\right]\left[d-k_i\right]$$

from which we find

$$H_i(p) \ = \ \left[\frac{p-v}{P-v}\right]^{\frac{d-k_1}{k_1-k_2-d}}$$

while i's revenues are [P-v][d0-k_i].

If we assume that firms play according to the mixed-strategy equilibrium whenever k_2 < d0 < k_1+k_2, at the investment stage we have the following expected profits (again firms are ordered such that $k_1 \leq k_2$):

$$\pi_1 = \left[P-v\right]\left\{\int_{k_1}^{k_2} k_1 dG(d) + \int_{k_2}^{k_1+k_2}\left[d-k_2\right]dG(d)\right\} + \int_{k_1+k_2}^{\bar{d}}\left[w-v\right]k_1 dG(d) - ck_1$$

$$\pi_2 = \left[P-v\right]\left\{\int_{k_1}^{k_2}\left[d-k_1\right]dG(d) + \int_{k_2}^{k_1+k_2}\left[d-k_1\right]dG(d)\right\} + \int_{k_1+k_2}^{\bar{d}}\left[w-v\right]k_2 dG(d) - ck_2$$

From these expressions we get the first-order conditions for optimal capacities:

$$\frac{\partial \pi_1}{\partial k_1} = \left[P-v\right]\left[G(k_2)-G(k_1)-k_1 G'(k_1)\right] + \left[w-v\right]\left[1-G(k_1+k_2)\right] - \left[w-P\right]k_1 G'(k_1+k_2) - c = 0$$

$$\frac{\partial \pi_2}{\partial k_2} = \left[w-v\right]\left[1-G(k_1+k_2)\right] - \left[w-P\right]k_2 G'(k_1+k_2) - c = 0$$

When P = v, the equilibrium is symmetric. When P > v, we can apply a similar argument to that above to demonstrate that there does not exist a symmetric equilibrium. When P = w the second first-order condition implies that industry capacity is at its first-best level. Consequently, in this equilibrium there is never over-investment, and whenever P < w there is under-investment.

APPENDIX 3: Equilibrium conditions in the multi-technology duopoly model

Assume that, in events in which multiple pure-strategy outcomes exists, with equal probability either of the two alternative equilibrium outcomes result. Fix t, and order firms such that k_1^t ✂ k_2^t. For t = T, we have the following expressions for expected profits earned on the technology T capacities:

$$\pi_1^T = \left[P - v_T\right]\left\{\int_{K^{T-1}+k_1^T}^{K^{T-1}+k_2^T} k_1^T dG(d) + \int_{K^{T-1}+k_2^T}^{K^T} \left\{ \tfrac{1}{2} k_1^T + \tfrac{1}{2}\left[d - K^{T-1} - k_2^T\right]\right\} dG(d)\right\} + \left[w - v_T\right]\int_{K^T}^{\bar{d}} k_1^T dG(d) - c_T k_1^T$$

$$\pi_2^T = \left[P - v_T\right]\left\{\int_{K^{T-1}+k_1^T}^{K^{T-1}+k_2^T} \left[d - K^{T-1} - k_1^T\right] dG(d) + \int_{K^{T-1}+k_2^T}^{K^T} \left\{ \tfrac{1}{2} k_2^T + \tfrac{1}{2}\left[d - K^{T-1} - k_1^T\right]\right\} dG(d)\right\} + \left[w - v_T\right]\int_{K^T}^{\bar{d}} k_2^T dG(d) - c_T k_2^T$$

For t = 1,2,…,T-1, the corresponding expressions are

$$\pi_1^t = \left[v_{t+1} - v_t\right]\left\{\int_{K^{t-1}+k_1^t}^{K^{t-1}+k_2^t} k_1^t dG(d) + \int_{K^{t-1}+k_2^t}^{K^t} \left\{ \tfrac{1}{2} k_1^t + \tfrac{1}{2}\left[d - K^{t-1} - k_2^t\right]\right\} dG(d) + \int_{K^t}^{K^t+\underline{k}^{t+1}} k_1^t dG(d)\right\}$$
$$+ \sum_{\tau=t}^{T-2}\left[v_{\tau+2} - v_t\right]\int_{K^\tau+\underline{k}^{\tau+1}}^{K^{\tau+1}+\underline{k}^{\tau+2}} k_1^t dG(d) + \left[P - v_t\right]\int_{K^{T-1}+\underline{k}^T}^{K^T} k_1^t dG(d) + \left[w - v_t\right]\int_{K^T}^{\bar{d}} k_1^t dG(d) - c_t k_1^t$$

$$\pi_2^t = \left[v_{t+1} - v_t\right]\left\{\int_{K^{t-1}+k_1^t}^{K^{t-1}+k_2^t} \left[d - K^{t-1} - k_1^t\right] dG(d) + \int_{K^{t-1}+k_2^t}^{K^t} \left\{ \tfrac{1}{2} k_2^t + \tfrac{1}{2}\left[d - K^{t-1} - k_1^t\right]\right\} dG(d) + \int_{K^t}^{K^t+\underline{k}^{t+1}} k_2^t dG(d)\right\}$$
$$+ \sum_{\tau=t}^{T-2}\left[v_{\tau+2} - v_t\right]\int_{K^\tau+\underline{k}^{\tau+1}}^{K^{\tau+1}+\underline{k}^{\tau+2}} k_2^t dG(d) + \left[P - v_t\right]\int_{K^{T-1}+\underline{k}^T}^{K^T} k_2^t dG(d) + \left[w - v_t\right]\int_{K^T}^{\bar{d}} k_2^t dG(d) - c_t k_2^t$$

where $\underline{k}^{\tau} = \min\{k_1^{\tau}, k_2^{\tau}\}$.

No profits are earned on the technology t capacity when demand is such that it may be covered by technologies with lower operating costs. Also in the case when technology t is marginal, but t-residual demand is smaller than the technology t capacity of firm 1, are there no profits earned on this technology. When t-residual demand cannot be covered by firm 1's technology t capacity, but is less than firm 2's technology t capacity, firm 1 is despatched with its entire technology t capacity, firm 2 covers residual demand and both firms are paid at the marginal operating cost of technology t+1 (the first term in the above profits expressions). When t-residual demand exceeds the technology t capacity of firm 2, with equal probabilities firms are either despatched with all of their technology t capacity or only supplies residual demand (the second term). When some other, more flexible technology is marginal, firms' payment on their technology t capacity is according to the corresponding equilibrium price (the remaining terms).

Note that the profit expressions for the two firms are identical, except for the event in which demand exceeds the capacity of the t-1 low-cost technologies and t-residual demand is larger than the technology t capacity of firm 1 but smaller than the technology t capacity of firm 2. In this event firm 1 bids low, and is despatched with its entire capacity, while firm 2 bids high and only supplies residual demand.

For t = 1,2,…,T, the first-order conditions for profit-maximising capacity choices may be written:

$$\frac{\partial \pi_i}{\partial k_i^t} = \sum_{\tau=1}^{T} \frac{\partial \pi_i^{\tau}}{\partial k_i^t} = 0, i = 1,2.$$

For t = T, we have for firm 1 (the firm with the smaller technology t capacity)

$$\frac{\partial \pi_1^T}{\partial k_1^T} = \left[P - v_T\right]\left[\tfrac{1}{2}G(K^T) + \tfrac{1}{2}G(K^{T-1} + k_2^T) - G(K^{T-1} + k_1^T) - k_1^T G'(K^{T-1} + k_1^T)\right]$$
$$+\left[w - v_T\right]\left[1 - G(K^T)\right] - \left[w - P\right]k_1^T G'(K^T) - c_T$$

$$\left.\frac{\partial \pi_1^{\tau}}{\partial k_1^T}\right|_{\tau<T} = -\left[P - v_T\right]k_1^{\tau} G'(K^{T-1} + k_1^T) - \left[w - P\right]k_1^{\tau} G'(K^T)$$

Similarly, for firm 2 (the firm with the larger technology t capacity)

$$\frac{\partial \pi_2^T}{\partial k_2^T} = \left[P - v_T\right]\left[\tfrac{1}{2} G(K^T) - \tfrac{1}{2} G(K^{T-1} + k_2^T) - \tfrac{1}{2} k_1^T G'(K^{T-1} + k_1^T)\right]$$
$$+ \left[w - v_T\right]\left[1 - G(K^T)\right] - \left[w - P\right] k_1^T G'(K^T) - c_T$$

$$\left.\frac{\partial \pi_1^\tau}{\partial k_1^T}\right|_{\tau < T} = -\left[w - P\right] k_2^\tau G'(K^T)$$

Consequently, for t = T, we may write the first-order conditions for optimal capacities as follows:

$$\frac{\partial \pi_1}{\partial k_1^T} = \left[P - v_T\right]\left[\tfrac{1}{2} G(K^T) + \tfrac{1}{2} G(K^{T-1} + k_2^T) - G(K^{T-1} + k_1^T) - K_1^T G'(K^{T-1} + k_1^T)\right]$$
$$+ \left[w - v_T\right]\left[1 - G(K^T)\right] - \left[w - P\right] K_1^T G'(K^T) - c_T = 0$$

$$\frac{\partial \pi_2}{\partial k_2^T} = \left[P - v_T\right]\left[\tfrac{1}{2} G(K^T) - \tfrac{1}{2} G(K^{T-1} + k_2^T) - \tfrac{1}{2} k_1^T G'(K^{T-1} + k_2^T)\right]$$
$$+ \left[w - v_T\right]\left[1 - G(K^T)\right] - \left[w - P\right] K_2^T G'(K^T) - c_T = 0$$

For t < T, we have for firm 1

$$\left.\frac{\partial \pi_1^\tau}{\partial k_1^t}\right|_{\tau < t} = -\left\{\sum_{s=t}^{T-1}\left[v_{s+1} - v_s\right] G'(K^{s-1} + \underline{k}^s) + \left[P - v_T\right] G'(K^{T-1} + \underline{k}^T) + \left[w - P\right] G'(K^T)\right\} k_1^\tau$$

$$\frac{\partial \pi_1^t}{\partial k_1^t} = \left[v_{t+1} - v_t\right]\left[G(K^t + \underline{k}^{t+1}) - \tfrac{1}{2} G(K^t) + \tfrac{1}{2} G(K^{t-1} + k_2^t) - G(K^{t-1} + k_1^t)\right]$$
$$+ \sum_{s=t+1}^{T-1}\left[v_{s+1} - v_t\right]\left[G(K^s + \underline{k}^{s+1}) - G(K^{s-1} + \underline{k}^s)\right]$$
$$+ \left[P - v_t\right]\left[G(K^T) - G(K^{T-1} + \underline{k}^T)\right] + \left[w - v_t\right]\left[1 - G(K^T)\right]$$
$$- \left\{\sum_{s=t}^{T-1}\left[v_{s+1} - v_s\right] G'(K^{s-1} + \underline{k}^s) + \left[P - v_T\right] G'(K^{T-1} + \underline{k}^T) + \left[w - P\right] G'(K^T)\right\} k_1^t - c_t$$

$$\left.\frac{\partial\pi_1^\tau}{\partial k_1^t}\right|_{\tau>t} = -\left\{\sum_{s=\tau}^{T-1}\left[v_{s+1}-v_s\right]G'(K^{s-1}+\underline{k}^s)+\left[P\text{-}v_T\right]G'(K^{T-1}+\underline{k}^T)+\left[w-P\right]G'(K^T)\right\}k_1^\tau$$

$$-\left[v_{\tau+1}-v_\tau\right]\begin{cases} \tfrac{1}{2}\left[G(K^\tau)-G(K^{\tau-1}+\overline{k}^\tau)\right]-\tfrac{1}{2}G'(K^{\tau-1}+\overline{k}^\tau)k_1^\tau & \text{if } k_1^\tau=\underline{k}^\tau \\ \tfrac{1}{2}G(K^\tau)+\tfrac{1}{2}G(K^{\tau-1}+\overline{k}^\tau)-G(K^{\tau-1}+\underline{k}^\tau) & \\ \quad -G'(K^{\tau-1}+\underline{k}^\tau)k_1^\tau+\tfrac{1}{2}G'(K^{\tau-1}+\overline{k}^\tau)k_1^\tau & \text{if } k_1^\tau=\overline{k}^\tau \end{cases}$$

where $v_{T+1}=P$ and $\overline{k}^\tau=\max\{k_1^\tau,k_2^\tau\}$.

The corresponding results for firm 2 are

$$\left.\frac{\partial\pi_2^\tau}{\partial k_2^t}\right|_{\tau<t} = -\left\{\sum_{s=t+1}^{T-1}\left[v_{s+1}-v_s\right]G'(K^{s-1}+\underline{k}^s)+\left[P\text{-}v_T\right]G'(K^{T-1}+\underline{k}^T)+\left[w-P\right]G'(K^T)\right\}k_2^\tau$$

$$\begin{aligned}\frac{\partial\pi_1^t}{\partial k_2^t} = {} & \left[v_{t+1}\text{-}v_t\right]\left[G(K^t+\underline{k}^{t+1})-\tfrac{1}{2}G(K^t)-\tfrac{1}{2}G(K^{t-1}+k_2^t)-\tfrac{1}{2}k_1^tG'(K^{t-1}+k_2^t)\right] \\ & +\sum_{s=t+1}^{T\text{-}1}\left[v_{s+1}\text{-}v_t\right]\left[G(K^s+\underline{k}^{s+1})-G(K^{s-1}+\underline{k}^s)\right] \\ & +\left[P\text{-}v_t\right]\left[G(K^T)-G(K^{T-1}+\underline{k}^T)\right]+\left[w\text{-}v_t\right]\left[1-G(K^T)\right] \\ & -\left\{\sum_{s=t+1}^{T-1}\left[v_{s+1}-v_s\right]G'(K^{s-1}+\underline{k}^s)+\left[P-v_T\right]G'(K^{T-1}+\underline{k}^T)+\left[w-P\right]G'(K^T)\right\}k_2^t\text{ - }c_t\end{aligned}$$

$$\left.\frac{\partial\pi_2^\tau}{\partial k_2^t}\right|_{\tau>t} = -\left\{\sum_{s=\tau}^{T-1}\left[v_{s+1}-v_s\right]G'(K^{s-1}+\underline{k}^s)+\left[P\text{-}v_T\right]G'(K^{T-1}+\underline{k}^T)+\left[w-P\right]G'(K^T)\right\}k_1^\tau$$

$$-\left[v_{\tau+1}-v_\tau\right]\begin{cases} \tfrac{1}{2}\left[G(K^\tau)-G(K^{\tau-1}+\overline{k}^\tau)\right]-\tfrac{1}{2}G'(K^{\tau-1}+\overline{k}^\tau)k_1^\tau & \text{if } k_1^\tau=\underline{k}^\tau \\ \tfrac{1}{2}G(K^\tau)+\tfrac{1}{2}G(K^{\tau-1}+\overline{k}^\tau)-G(K^{\tau-1}+\underline{k}^\tau) & \\ \quad -G'(K^{\tau-1}+\underline{k}^\tau)k_1^\tau+\tfrac{1}{2}G'(K^{\tau-1}+\overline{k}^\tau)k_1^\tau & \text{if } k_1^\tau=\overline{k}^\tau \end{cases}$$

If an equilibrium exists in which one firm has a smaller capacity of each technology (i.e. k_1^t ✄ k_2^t, all t) the first-order conditions may be written

$$
\begin{aligned}
\frac{\partial \pi_1}{\partial k_1^t} \;=\; & \sum_{s=t}^{T\text{-}1}\left[v_{s+1} \text{-} v_t\right]\left[G(K^s + k_1^{s+1}) - G(K^{s-1} + k_1^s)\right] \\
& + \left[P \text{-} v_t\right]\left[G(K^T) - G(K^{T-1} + k_1^T)\right] + \left[w \text{-} v_t\right]\left[1 - G(K^T)\right] \\
& + \sum_{s=t+1}^{T-1} \tfrac{1}{2}\left[v_{s+1} - v_s\right] k_1^s G'(K^{s-1} + k_2^s) + \tfrac{1}{2}\left[P - v_T\right] k_1^T G'(K^{T-1} + k_2^T) \\
& - \sum_{s=t}^{T-1} \tfrac{1}{2}\left[v_{s+1} - v_s\right]\left[G(K^s) - G(K^{s-1} + k_2^s)\right] - \tfrac{1}{2}\left[P - v_T\right]\left[G(K^T) - G(K^{T-1} + k_2^T)\right] \\
& - \sum_{s=t}^{T-1}\left[v_{s+1} - v_s\right] K_1^s G'(K^{s-1} + k_1^s) - \left[P - v_T\right] K_1^T G'(K^{T-1} + k_1^T) - \left[w - P\right] K_1^T G'(K^T) \text{-} c_t = 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \pi_2}{\partial k_2^t} \;=\; & \sum_{s=t}^{T\text{-}1}\left[v_{s+1} \text{-} v_t\right]\left[G(K^s + k_1^{s+1}) - G(K^{s-1} + k_1^s)\right] \\
& + \left[P \text{-} v_t\right]\left[G(K^T) - G(K^{T-1} + k_1^T)\right] + \left[w \text{-} v_t\right]\left[1 - G(K^T)\right] \\
& - \sum_{s=t}^{T-1} \tfrac{1}{2}\left[v_{s+1} - v_s\right] k_1^s G'(K^{s-1} + k_2^s) - \tfrac{1}{2}\left[P - v_T\right] k_1^T G'(K^{T-1} + k_2^T) \\
& - \sum_{s=t}^{T-1}\left[v_{s+1} - v_s\right]\left[\tfrac{1}{2} G(K^s) + \tfrac{1}{2} G(K^{s-1} + k_2^s) - G(K^{s-1} + k_1^s)\right] \\
& - \left[P - v_T\right]\left[\tfrac{1}{2} G(K^T) + \tfrac{1}{2} G(K^{T-1} + k_2^T) - G(K^{T-1} + k_1^T)\right] \\
& - \sum_{s=t+1}^{T-1}\left[v_{s+1} - v_s\right] K_2^s G'(K^{s-1} + k_1^s) - \left[P - v_T\right] K_2^{T-1} G'(K^{T-1} + k_1^T) - \left[w - P\right] K_2^T G'(K^T) \text{-} c_t = 0
\end{aligned}
$$